Waves drift forces and motion response of moored tankers in bi- and multi-chromatic waves

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ABSTRACT

In regular wave groups (bi-chromatic waves) the wave drift forces acting on a tanker consist of a mean wave drift force, an oscillating wave drift force and a wave drift damping force. By means of model tests or 3-dimensional potential theory the quadratic transfer function of the wave drift force and the wave drift damping can be determined. For the low frequency surge motions of a linearly moored tanker exposed to regular wave groups the computational procedure is shown. To validate the computational procedure model tests have been carried out. The computational code is extended to irregular head waves (multi-chromatic waves) and verified by means of model tests.

NOMENCLATURE

- a_{11} = added mass in surge direction
- $A(t)$ = wave envelope
- B_{11} = viscous still water damping
- B_1 = mean wave drift damping coefficient
- c_{11} = spring constant in surge direction
- D_{ij} = amplitude of wave drift damping for the wave frequencies ω_i and ω_j
- M = mass of the vessel
- P_{ij}, Q_{ij} = real and imaginary part of the transfer function of the wave drift force for the wave frequencies ω_i and ω_j
- S_{x_1} = spectral density of the low frequency surge motion
- S_{X_1} = spectral density of the wave drift force in surge direction
- T_{ij} = amplitude of the transfer function of the wave drift damping coefficient for the wave frequencies ω_i and ω_j
- t = time
- x_1 = low frequency displacement in surge direction
- $\dot{x}_1$ = low frequency velocity in surge direction
- $\ddot{x}_1$ = low frequency acceleration in surge direction
- $X_1(t)$ = wave drift force
- $X_{1t}(t)$ = total wave drift force
- X_1 = mean wave drift force
- X_{1a} = amplitude of oscillating wave drift force
- ε = phase angle
- ζ_i = amplitude of the i-th wave component
- ω_i = frequency of the i-th wave component
- μ = frequency of low frequency part of the wave drift forces
- μ_1 = natural frequency of the system in surge direction
- σ_x = standard deviation of low frequency surge motion
- $\zeta_{w1/3}$ = significant wave height
- $\bar{T}_1$ = mean wave period
- T_i = period of the i-th wave component

INTRODUCTION

A tanker moored in irregular head waves will have a mean offset and performs high (= wave) frequency surge, heave and pitch motions and slowly oscillating surge motions. The high frequency motions are caused by the high frequency wave loads, while the mean offset and the slowly oscillating motions are caused by the wave drift forces.

It is assumed that the wave drift force consists of a mean part and an oscillating part. The mean wave drift force is responsible for the offset, while the oscillating part causes the slowly oscillating motion. From the point of view of theory the mean wave drift force and the oscillating part are defined under the condition that the floating body performs high frequency (first order) motions about the mean position. By means of the direct pressure integration method as given by for instance Pinkster [1] the matrix of the quadratic transfer function of the wave drift forces can be computed.

In reality, however, the tanker does not perform high frequency motions about the mean position. The high frequencies are present while the tanker performs slowly oscillating motions. Due to the slowly oscillating motions the high frequency motions will be speed dependent. As a result the wave drift forces are speed dependent. Taking into account the speed effect Wichers [2], [3] has shown that the total wave drift force in irregular waves consists of a mean part, a oscillating part and a wave drift damping part.

In the theory the wave drift coefficient can consist of a constant and an oscillating part. In this paper it will be shown that the influence of the oscillating part of the wave drift damping on the low frequency motions is negligibly small. To apply the results a computational code has been developed to predict the low frequency surge motions of a tanker moored in head waves. The computational procedure has been applied first to the tanker exposed to regular wave groups and then extended to irregular waves.

The computational code is validated by means of model tests. The wave drift damping coefficient has been experimentally determined and computed.

For the computations the 3-dimensional potential theory with low forward speed has been applied. The theory is shown by Wichers [2] and Hermans and Huijsmans [4].

THEORY WAVE DRIFT FORCES

Wave Drift Force For Zero Speed

In order to arrive at the theory of the speed dependent wave drift forces first the derivation will be given for the wave drift forces for zero speed as treated in ref. [1]. The behaviour of the drift forces in waves can be elucidated by looking at the general expression for the drift forces in a wave train consisting of two regular sinusoidal waves with frequencies ω_1 and ω_2 and amplitudes ζ_1 and ζ_2.

The wave elevation can be written as follows:

$$\zeta(t) = \sum_{i=1}^{2} \zeta_i \sin(\omega_i t+\varepsilon_i) = \zeta_1 \sin(\omega_1 t+\varepsilon_1) + \zeta_2 \sin(\omega_2 t+\varepsilon_2) \tag{1}$$

For small differences between ω_1 and ω_2 a schematic representation of the wave train is shown in Fig. 1. Such a wave train will be called a regular wave group. This type of wave train is characterized by a periodic variation of the wave envelope. The frequency associated with the envelope is equal to $\Delta\omega = \omega_1-\omega_2$ being the difference frequency of the regular wave components.

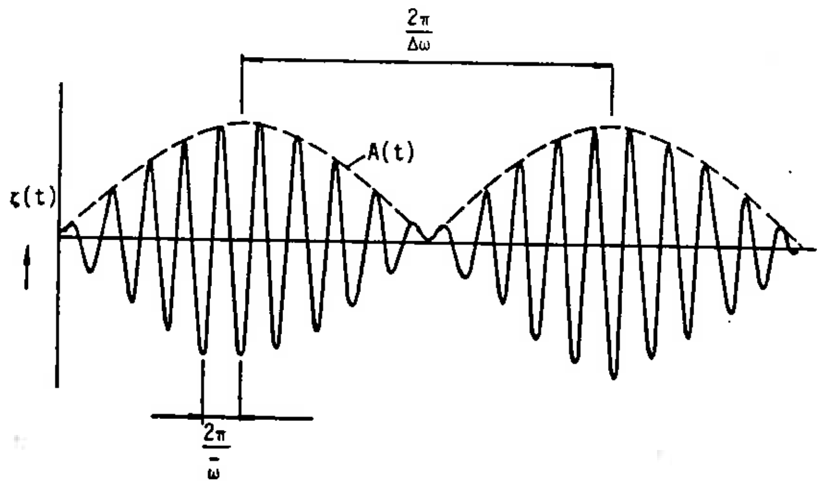

Fig. 1 Regular wave group

We will write the wave elevation in amplitude modulated form:

$$\zeta(t) = A(t)\sin(\bar{\omega}t + \bar{\varepsilon}) \quad (2)$$

in which:

$\bar{\omega} = (\omega_1 + \omega_2)/2$

$\bar{\varepsilon} = (\varepsilon_1 + \varepsilon_2)/2$

It can be shown that the envelope becomes:

$$A(t) = \left[\sum_{i=1}^{2}\sum_{j=1}^{2} \zeta_i\zeta_j \cdot \cos\left((\omega_i-\omega_j)t + (\varepsilon_i-\varepsilon_j)\right)\right]^{1/2} \quad (3)$$

The square of the envelope is:

$$A^2(t) = \sum_{i=1}^{2}\sum_{j=1}^{2} \zeta_i\zeta_j \cdot \cos\left((\omega_i-\omega_j)t + (\varepsilon_i-\varepsilon_j)\right) \quad (4)$$

A quantity which is a quadratic function of the wave amplitude, in this case the wave drift force, will be:

$$X_1(t) = \sum_{i=1}^{2}\sum_{j=1}^{2} \zeta_i\zeta_j P_{ij}\cos\left((\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)\right) + \sum_{i=1}^{2}\sum_{j=1}^{2} \zeta_i\zeta_j Q_{ij}\sin\left((\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)\right) \quad (5)$$

in which P_{ij} and Q_{ij} are quadratic transfer functions dependent on two frequencies ω_i and ω_j. Generally P_{ij} and Q_{ij} are computed so that the following relations exist:

$P_{ij} = P_{ji}$

$Q_{ij} = -Q_{ji}$

P_{ij} is that part of the quadratic transfer function which expresses the component of the drift force which is in-phase with the square of the wave envelope and Q_{ij} expresses the quadrature part of the drift force. For the regular wave group the wave drift force is:

$$X_1(t) = \zeta_1^2 P_{11} + \zeta_2^2 P_{22} + \zeta_1\zeta_2(P_{12}+P_{21}) \cdot \cos\left((\omega_1-\omega_2)t + (\varepsilon_1-\varepsilon_2)\right) + \zeta_1\zeta_2(Q_{12}-Q_{21}) \cdot \sin\left((\omega_1-\omega_2)t + (\varepsilon_1-\varepsilon_2)\right) \quad (6)$$

The formulation shows that the drift force contains several components. The first two are constant parts corresponding to the mean drift force in each of the regular wave components separately. The third and fourth parts are low frequency varying components which arise through the combined presence of the two regular wave components in the wave group.

The quadratic transfer function of the wave drift force in terms of amplitudes and phase angles are defined as:

$$T_{ij} = T(\omega_i,\omega_j) = \left(P^2(\omega_i,\omega_j) + Q^2(\omega_i,\omega_j)\right)^{1/2} \quad (7)$$

= quadratic transfer function of the amplitude of the wave drift force

$$\varepsilon_{ij} = \arctan - \frac{Q(\omega_i,\omega_j)}{P(\omega_i,\omega_j)} \quad (8)$$

= phase angle between the low frequency part of the second order force relative to the low frequency part of the square of the wave elevation.

Using the mentioned definition of the quadratic transfer function the wave drift force for the regular wave group can be written as:

$$X_1(t) = \sum_{i=1}^{2}\sum_{j=1}^{2} \zeta_i\zeta_j T_{ij} \cdot \cos\left((\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)+\varepsilon_{ij}\right) \quad (9)$$

In irregular waves the wave drift force is:

$$X_1(t) = \sum_{i=1}^{N}\sum_{j=1}^{N} \zeta_i\zeta_j T_{ij} \cdot \cos\left((\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)+\varepsilon_{ij}\right) \quad (10)$$

The quadratic transfer function P_{ij} and Q_{ij} for zero speed can be computed by means of the direct pressure integration method as given in ref. [1].

Total Wave Drift Force

If the total wave drift force acting on a moored tanker exposed to a regular wave group will be considered then the low frequency surge velocity of the vessel has to be taken into account as is shown below, see ref. [2]:

$$X_{1t}(t) = X_1(t) + \frac{\partial X_1(t)}{\partial \dot{x}_1}\dot{x}_1 =$$
$$= \zeta_1^2 T_{11} + \zeta_2^2 T_{22} + 2\zeta_1\zeta_2 T_{12}\cdot$$
$$\cdot\cos\left(\Delta\omega_{12}t+\Delta\varepsilon_{12}+\varepsilon_{12}\right) +$$
$$+ \zeta_1^2 D_{11}\dot{x}_1 + \zeta_2^2 D_{22}\dot{x}_1 +$$
$$+ 2\zeta_1\zeta_2 D_{12}\cos\left(\Delta\omega_{12}t+\Delta\varepsilon_{12}+\varepsilon_{12}\right)\dot{x}_1 \quad (11)$$

in which:

$D_{11} = \partial T_{11}/\partial \dot{x}_1$

$D_{22} = \partial T_{22}/\partial \dot{x}_1$

$D_{12} = \partial T_{12}/\partial \dot{x}_1$

The total wave drift force can be split into a wave drift force and a wave drift damping force. The wave drift damping force contains several components. The coefficient of the first two terms correspond to the mean wave drift damping in each of the regular wave components separately. The third term stands for the low frequency varying part of the wave drift damping force.

In irregular waves the total wave drift force will be:

$$X_{1t} = \sum_{i=1}^{N}\sum_{j=1}^{N} \zeta_i\zeta_j T_{ij}\cdot$$
$$\cdot\cos\left((\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)+\varepsilon_{ij}\right) +$$
$$+ \sum_{i=1}^{N}\sum_{j=1}^{N} \zeta_i\zeta_j D_{ij}\dot{x}_1\cdot$$
$$\cdot\cos\left((\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)+\varepsilon_{ij}\right) \quad (12)$$

Stability Of The Solution

The effect of the damping will play an important role for the condition that the natural frequency of the moored vessel will correspond to the frequency of the wave group:

$$\Delta\omega_{12} = \mu_1 = \sqrt{\frac{c_{11}}{M+a_{11}(\mu_1)}} \quad (13)$$

Assuming that the linear viscous damping B_{11} is known, see ref. [2], the equation of low frequency motion can be written as follows:

$$\left(M+a_{11}(\mu_1)\right)\ddot{x}_1 + \left(B_{11}(\mu_1) - \zeta_1^2 D_{11} +\right.$$
$$- \zeta_2^2 D_{22} - 2\zeta_1\zeta_2 D_{12}\cdot$$
$$\left.\cdot\cos(\mu_1 t+\Delta\varepsilon_{12}+\varepsilon_{12})\right)\dot{x}_1 + c_{11}x_1 =$$
$$= \zeta_1^2 T_{11} + \zeta_2^2 T_{22} + 2\zeta_1\zeta_2 T_{12}\cdot$$
$$\cdot\cos(\mu_1 t+\Delta\varepsilon_{12}+\varepsilon_{12}) \quad (14)$$

The damping term contains linear coefficients and a low frequency oscillating coefficient. Due to the low frequency oscillating coefficient the value of the damping coefficient can be both positive and negative. Since a negative damping in the equation of motion can cause an unstable solution attention has to be paid to the magnitude of the oscillating damping coefficient with regard to the linear damping coefficients.

In order to judge the influence of the low frequency oscillating coefficient on the stability of the solution the equation of motion is simplified as follows:

$$m\ddot{x}_1 + f(t)\dot{x}_1 + cx_1 = a\cos\mu_1 t \quad (15)$$

in which:

$f(t) = b_1 + b_2\cos\mu_1 t$

$a\cos\mu_1 t$ = oscillating part of the wave drift force.

Starting from the equation of motion for the tanker

$$m\ddot{x}_1 + f(t)\dot{x}_1 + cx_1 = 0 \quad (16)$$

and multiplying this equation by $\dot{x}_1$, we obtain:

$$m\ddot{x}_1\dot{x}_1 + f(t)\dot{x}_1^2 + cx_1\dot{x}_1 = 0 \quad (17)$$

or in terms of energy:

$$\frac{d}{dt}\left(\tfrac{1}{2}m\dot{x}_1^2 + \tfrac{1}{2}cx_1^2\right) = -f(t)\dot{x}_1^2 \quad (18)$$

This means that the decrease of the total energy corresponds to $f(t)\dot{x}_1^2$. The system is called unstable if the term $f(t)$ is negative. In order to judge the stability the sign of $f(t)$ will be studied.

Since $\zeta_i^2 + \zeta_j^2 \geq 2\zeta_i\zeta_j$ and assuming that the magnitude of the oscillating damping coefficient can be approximated as follows:

$$D_{ij}(\omega_i,\omega_j) \sim D\left((\omega_i+\omega_j)/2,(\omega_i+\omega_j)/2\right)$$

and because the still water damping has to be added to the linear parts of the wave damping coefficients it can be concluded that the low frequency oscillating damping coefficient will be smaller than the linear damping coefficient. Therefore the sign of f(t) will be positive for all values of t.

Contribution Of The Oscillating Wave Drift Damping

Besides the stability of the solution also attention will be paid to the contribution of the oscillating damping to the motion. For the considered equation of motion

$$m\ddot{x}_1 + (b_1 + b_2 \cos \mu_1 t)\dot{x}_1 + cx_1 = a \cos \mu_1 t \tag{19}$$

the following solution has been assumed:

$$x_1 = p_0 + \sum_{n=1}^{\infty} \left(p_n \cos(n\mu_1 t) + q_n \sin(n\mu_1 t)\right) \tag{20}$$

Substituting the solution in the equation of motion it can be proven that the oscillating coefficient hardly contributes to the motions.

The same conclusion can be drawn if the product of the oscillating damping coefficient and the low frequency velocity of the tanker in a wave group will be considered directly. The low frequency motion is assumed to be:

$$x(t) = x_{1a} \cos(\mu_1 t+\varepsilon) \tag{21}$$

in which:
x_{1a} = amplitude of the motion
ε = phase angle between oscillating drift force and the motion,

and the oscillating damping force may be written as:

$$X_{b0} = -b_2 \cos(\mu_1 t)\, x_{1a}\mu \sin(\mu_1 t+\varepsilon) = -\tfrac{1}{2} b_2 x_{1a}\mu_1 \left\{\sin(\varepsilon) + \left(\sin(2\mu_1 t)\cdot \cos(\varepsilon) + \cos(2\mu_1 t)\sin(\varepsilon)\right)\right\} \tag{22}$$

The oscillating damping force consists of a mean force and an oscillating force. The oscillating force acts with a double frequency. This double frequency, however, is beyond the resonance frequency and so the contribution will be negligibly small. Further, the constant part will affect the mean wave drift force slightly.

Neglecting the term with the oscillating coefficient the total wave drift force in a regular wave group will be:

$$X_{1t}(t) = \sum_{i=1}^{2}\sum_{j=1}^{2} \zeta_i\zeta_j T_{ij}\cdot \cos\left((\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)+\varepsilon_{ij}\right) + \sum_{i=1}^{2} \zeta_i^2 D_{ii}\dot{x}_1 \tag{23}$$

Following the mentioned procedure the total wave drift force in irregular waves will be:

$$X_{1t} = \sum_{i=1}^{N}\sum_{j=1}^{N} \zeta_i\zeta_j T_{ij}\cdot \cos\left((\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)+\varepsilon_{ij}\right) + \sum_{i=1}^{N} \zeta_i^2 D_{ii}\dot{x}_1 \tag{24}$$

COMPUTATION AND MODEL TESTS REGULAR WAVE GROUPS

Tanker And Mooring System

The computations and the model tests have been applied to a loaded 200 kDWT tanker moored in 82.5 m deep water. The tanker was linearly moored. The spring constant amounts to c_{11} = 17.7 tf/m. The particulars of the tanker are given in Table I. The body plan is shown in Fig. 2. The set-up of the mooring is given in Fig. 3.

Computation

The quadratic transfer function of the wave drift forces as presented by Pinkster [1] and the quadratic transfer function of the wave drift damping as given by Wichers [2] will be used for the computations. The transfer functions concern the fully loaded 200 kDWT in 82.5 m water depth and are presented in the Figs. 4, 5 and 6. The necessary viscous damping is obtained from the decay test in still water and amounts to B_{11} = 20.5 tf.s/m. The particulars of the wave groups as used for the computations are given in Table II.

Table I: Particulars and stability data of 200 kDWT tanker

Designation	Symbol	Unit	Magnitude
Length between perpendiculars	L_{PP}	m	310.00
Breadth	B	m	47.17
Depth	D	m	29.70
Draft	T	m	18.90
Displacement weight	Δ	tf	240,869
Centre of gravity above keel	$\overline{KG}$	m	13.32
Centre of buoyancy forward of section 10	$\overline{FB}$	m	6.60
Metacentric height	$\overline{GM}$	m	5.78
Longitudinal radius of gyration in air	k_{yy}	m	77.47
Waterplane coefficient	C_W	-	0.90
Midship section coefficient	C_M	-	0.95
Block coefficient	C_B	-	0.85

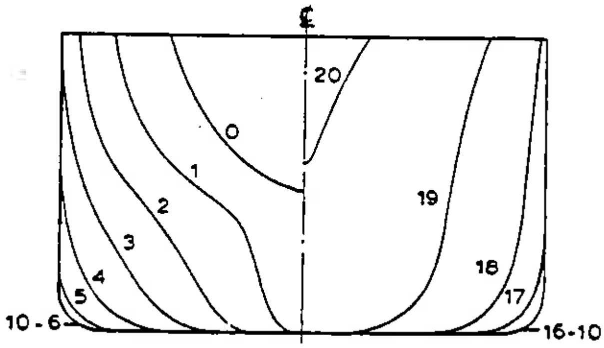

Fig. 2 Body plan of the 200 kDWT tanker

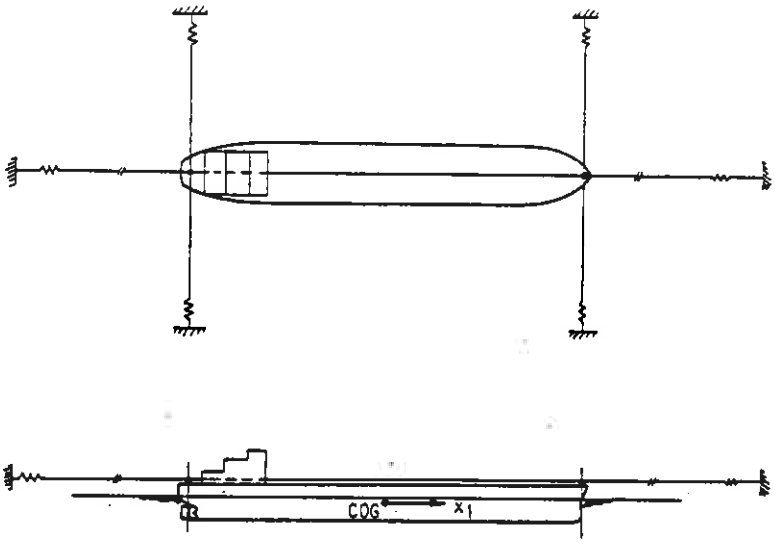

Fig. 3 Set-up of the linearly moored tanker

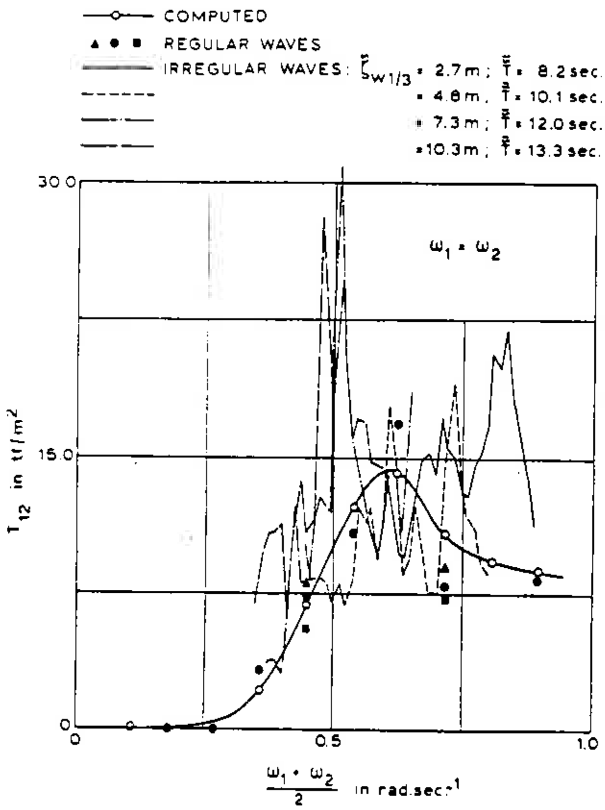

Fig. 4 Quadratic transfer function of the low frequency longitudinal drift force on the tanker [1]

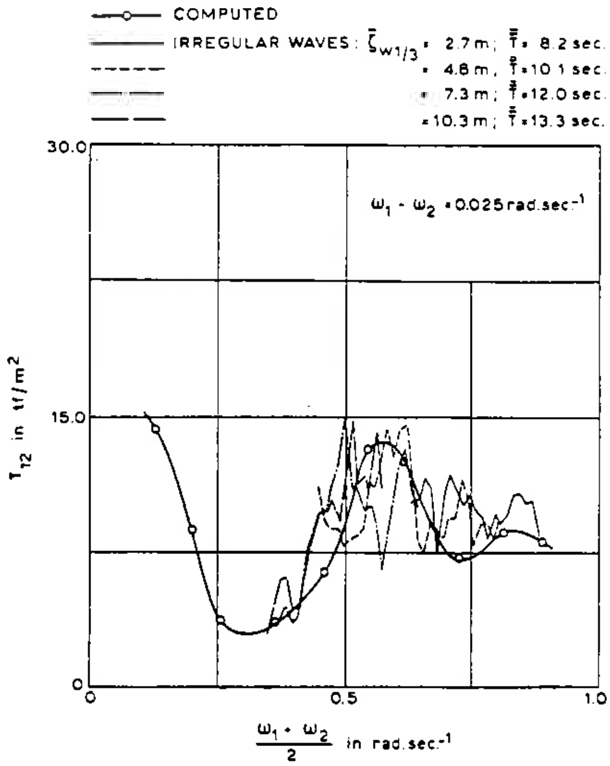

Fig. 5 Quadratic transfer function of the low frequency longitudinal drift force on the tanker [1]

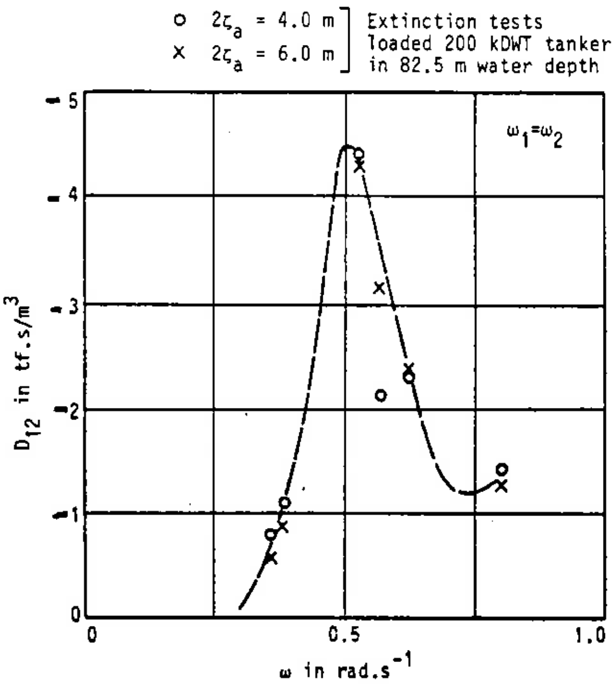

Fig. 6 Experimentally derived values of the wave drift damping quadratic transfer function [2]

Table II: Regular wave groups

Adjusted regular wave components:

Wave group No.	T_1 (s)	ω_1 (rad/s)	ζ_1 (m)	T_2 (s)	ω_2 (rad/s)	ζ_2 (m)
I	9.72	0.6465	0.73	10.11	0.6215	0.74
II	9.72	0.6465	1.38	10.11	0.6215	1.48
III	11.25	0.560	0.71	11.78	0.535	0.72
IV	11.25	0.560	1.32	11.78	0.535	1.34

Measured regular wave group:

Wave group No.	T (s)	$(\omega_1+\omega_2)/2$ (rad/s)	T_g (s)	$(\omega_1-\omega_2)$ (rad/s)	ζ_a (m)
I	9.9	0.6334	258	0.0243	1.42
II	9.9	0.6334	258	0.0243	2.85
III	11.5	0.546	251	0.0250	1.43
IV	11.5	0.546	251	0.0250	2.65

For the computation of the motions the following formulas are used:

$$x_1 = X_1/C_{11}$$

$$x_{1a} = X_{1a}/(B_{tot}(\omega_1-\omega 2)) \qquad (25)$$

in which:

x_1 = mean displacement

X_1 = mean wave drift force $= T_{11}(\omega_1)\zeta_1^2 + T_{22}(\omega_2)\zeta_2^2$

x_{1a} = motion amplitude

X_{1a} = oscillating wave drift force $= 2T_{12}\left((\omega_1+\omega_2)/2,\ (\omega_1-\omega_2)\right)\zeta_1\zeta_2$

Btot = total damping coefficient $= B_{11} + D_{11}(\omega_1)\zeta_1^2 + D_{22}(\omega_2)\zeta_2^2$

For the resonance frequency equation (25) corresponds to equation (28). The input data for the computations are presented in Table III. The results of the computations are given in Table IV.

Table III: Input data for computations in the regular wave groups

Wave group No.	Viscous damping (tf.s/m)	Wave drift damping (tf.s/m)	X_1 (tf)	X_{1a} (tf)
I	20.5	2.4	-15	13
II	20.5	9.1	-56	47
III	20.5	4.0	-13	14
IV	20.5	13.8	-45	48

Table IV: Motions in regular wave groups

Wave group No.	T (s)	$(\omega_1+\omega_2)/2$ (rad/s)	T_g (s)	$(\omega_1-\omega_2)$ (rad/s)	ζ_a (m)	Theory x (m)	Theory x_a (m)	Measured x (m)	Measured x_a (m)
I	9.9	0.6334	258	0.0243	1.42	-0.8	23.4	-3.0	24.8
II	9.9	0.6334	258	0.0243	2.85	-3.2	65.0	-6.3	49.5
III	11.5	0.546	251	0.0250	1.43	-0.7	23.0	-2.5	26.5
IV	11.5	0.546	251	0.0250	2.65	-2.5	56.0	-4.0	55.0

Model Tests

To validate the results of the motion computations model tests with the tanker exposed to regular wave groups were carried out. The tests were performed in the Wave and Current Laboratory of MARIN. The model scale was 82.5. All data presented were scaled to full scale according to Froude's law of similitude.

In order to generate the regular wave groups the wave generator was split as is shown in Fig. 7. One part of the wave flaps generates regular waves with frequency ω_1 and wave amplitude ζ_1, while the other wave flaps generate a regular wave with frequency ω_2 and wave amplitude ζ_2. The flaps of both wave generators were set under a small phase shift in order to generate the regular wave group. The adjusted regular wave groups are presented in Table II. Examples of typical wave groups are given in Figs. 8 and 9. For the regular wave groups the pre-tension in the transverse springs were slightly adjusted so that the natural period of the tanker in still water corresponds to the frequency of the regular wave group, see set-up in Fig. 3.

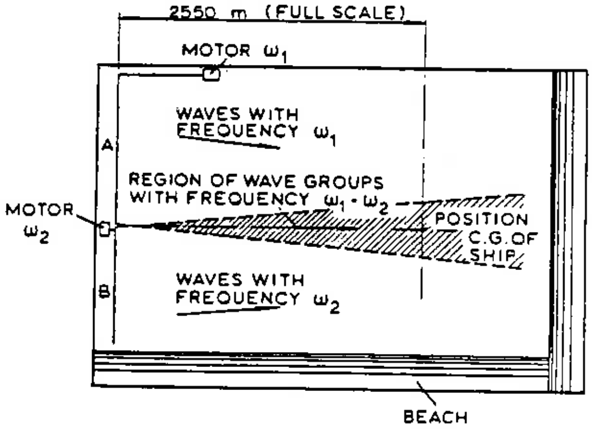

Fig. 7 Simulation of wave groups in the Wave and Current Laboratory

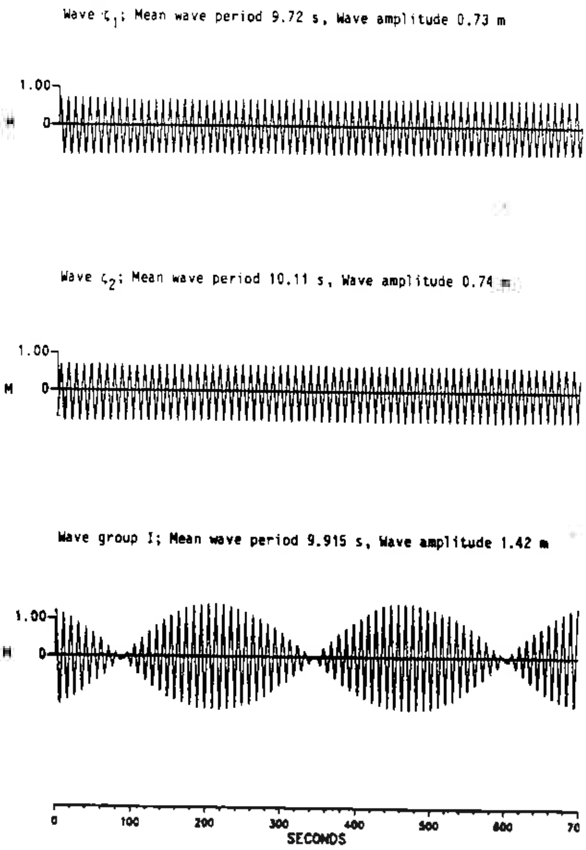

Fig. 8 Wave group I

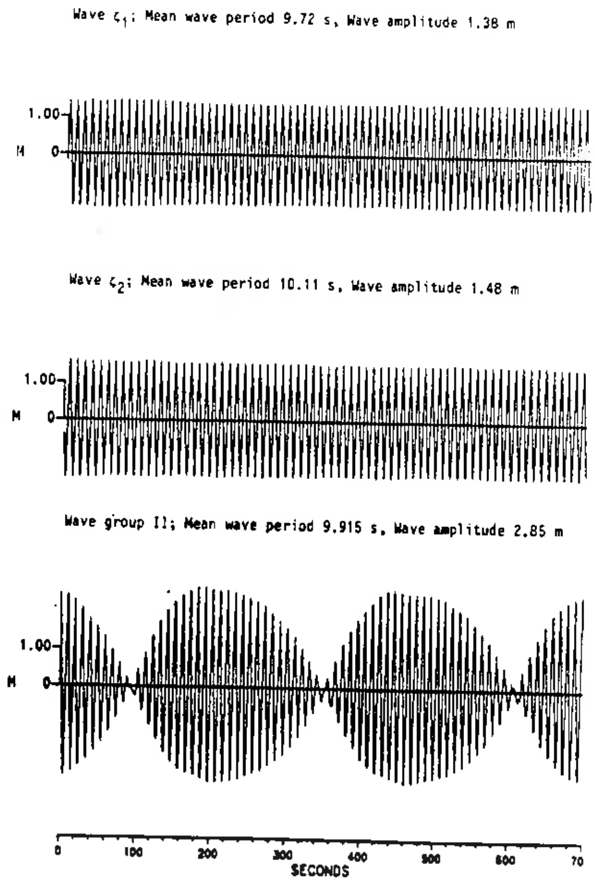

Fig. 9 Wave group II

The motions were measured by means of an optical measuring device. The measuring point corresponds to the centre of gravity (COG). Decay tests have been carried out in still water. Besides a decay curve in still water also extension tests have been carried out in a regular wave group. The decay curves are given in Fig. 10. The results of the measured motions of the tanker in the wave groups are given in Table IV.

Discussion Of Results

The damping is derived from Fig. 10 according to the following formulation:

$$B_{11} = \left(\sqrt{c_{11}(M+a_{11})}/\pi\right) \cdot (\ln_1 - \ln_{1+N})/N$$

in which:
M = 24,553 tf.s^2/m
a_{11} = 1,594 tf.s^2/m
N = number of oscillations.

The derived viscous damping amounts to B_{11} = 20.5 tf.s/m. The obtained still water damping agrees well with the data given in ref. [2].

For wave group I decay tests were carried out, see Fig. 9. Comparing the wave drift damping coefficients the following values were obtained:

Wave group No.	Computed (tf.s/m)	Measured (tf.s/m)
I	2.4	3.3
II	9.1	11.0

The measured results correspond reasonably well with the computed values. The results of the measured and calculated motions are given in Table IV. Considering the amplitudes of the motions a good agreement was found between model tests and computations. The mean displacements, however, were for the model tests somewhat larger than computed.

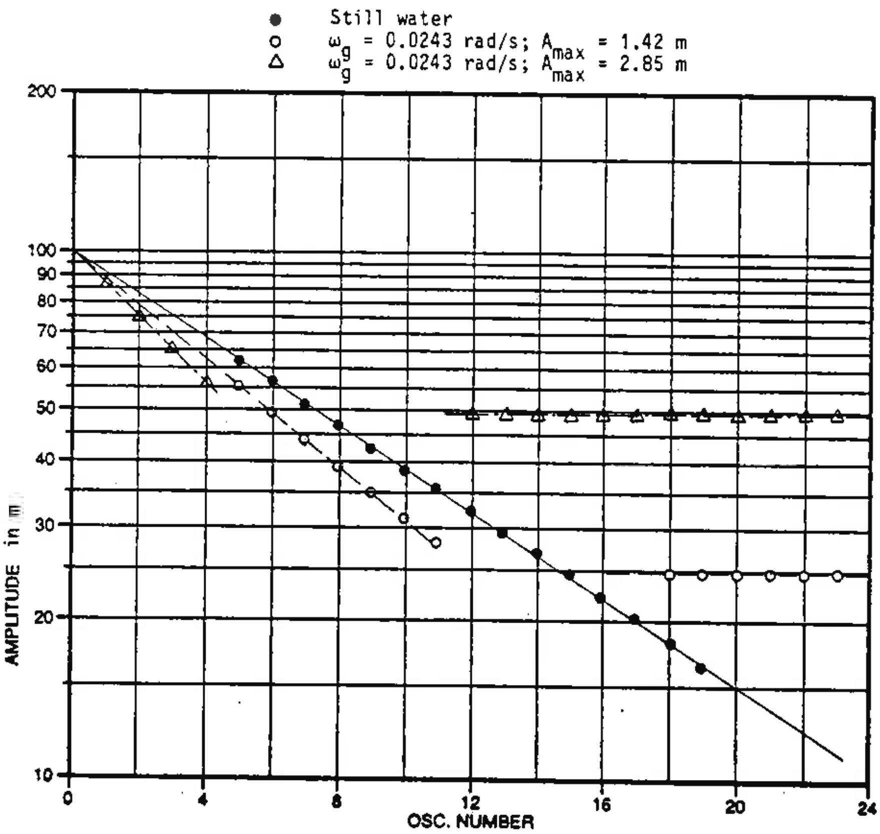

Fig. 10 Measured damping in still water and in regular wave groups

COMPUTATION AND MODEL TESTS IN IRREGULAR WAVES

Theory On The Frequency Domain Computations

The equation of the low frequency surge motion of a linearly moored tanker exposed to irregular head waves can be written as:

$$\left(M+a_{11}(\mu_1)\right)\ddot{x}_1 + B_{11}(\mu_1)\dot{x}_1 + + B_1\dot{x}_1 + c_{11}x_1 = X_1(t) \qquad (26)$$

in which:

$a_{11}(\mu_1)$ = added mass coefficient at the natural frequency μ_1
$B_{11}(\mu_1)$ = still water damping coefficient at the natural frequency μ_1
B_1 = mean wave drift damping coefficient
c_{11} = linear spring coefficient
$X_1(t)$ = wave drift force registration.

In the frequency domain the quantities considered are expressed in terms of spectral densities. Since the equation of motion is in a linear form, the spectral density of the low frequency surge motion can be written as:

$$S_{x_1}(\mu) = S_{X_1}(\mu)\left(\frac{x_{1a}}{X_{1a}}(\mu)\right)^2$$

while the variance of the low frequency surge motion will be:

$$\sigma^2_{x_1} = \int_0^\infty S_{X_1}(\mu)\left(\frac{x_{1a}}{X_{1a}}(\mu)\right)^2 d\mu \qquad (27)$$

where:

$S_{X_1}(\mu)$ = spectral density of the longitudinal wave drift force
$\frac{x_{1a}}{X_{1a}}(\mu)$ = surge amplitude per unit longitudinal wave drift force

$$= \frac{1}{\sqrt{(c_{11}-m_{11}\mu^2)^2 + (B_{11}+\bar{B}_1)^2\mu^2}} \qquad (28)$$

μ = frequency of low frequency part of the second order forces
m_{11} = $M + a_{11}(\mu_1)$

For the systems with a small damping, the response of the surge motion at the natural frequency dominates (has a peak). Therefore the spectral density can be kept constant over the frequency range. Following ref. [5] the variance reduces to the following form:

$$\sigma^2_{x_1} = S_{X_1}(\mu_1)\int_0^\infty \left(\frac{x_{1a}}{X_{1a}}(\mu)\right)^2 d\mu$$

which yields:

$$\sigma^2_{x_1} = \frac{\pi}{2(B_{11}+\bar{B}_1)c_{11}} S_{X_1}(\mu_1) \qquad (29)$$

where:

μ_1 = $\sqrt{c_{11}/m_{11}}$ =
= natural frequency of the system
$S_{X_1}(\mu_1)$ = spectral density of the wave drift force at frequency μ_1.

To solve equation (29) the input data have to be known. The still water damping coefficient can be read from Fig. 10. The mean wave drift damping coefficient B_1 in an irregular sea with N wave components can be determined as follows:

in series notation:

$$B_1 = \sum_{i=1}^{N} \zeta_i^2 \, D(\omega_i)$$

or in spectral notation: (30)

$$B_1 = 2\int_0^\infty S_\zeta(\omega)\, D(\omega)\, d\omega$$

in which:

$S_\zeta(\omega)$ = spectral density of the irregular sea.

The spectral density of the wave drift forces in an irregular sea state with spectral density $S(\omega)$ can be computed following ref. [5]:

$$S_{X_1}(\mu) = 8\int_0^\infty S_\zeta(\omega)\, S_\zeta(\omega+\mu)\cdot \left(T(\omega,\omega+\mu)\right)^2 d\omega \qquad (31)$$

in which:

$T(\omega,\omega+\mu)$ = amplitudes of the quadratic transfer function of the wave drift force dependent on ω and $\omega+\mu$.

Because the natural frequency in surge direction for a moored tanker is small the spectral density $S_{X_1}(\mu_1)$ will approximately be equal to $S_{X_1}(\mu=0)$, or:

$$S_{X_1}(\mu=0) = 8\int_0^\infty S^2_\zeta(\omega)\left(T(\omega,\omega)\right)^2 d\omega \qquad (32)$$

Tanker And Mooring System

The computations and model tests have been applied to the loaded 200 kDWT tanker moored in 206 m water depth. The tanker was moored by means of a linear spring constant c_{11} = 13.6 tf/m. The particulars of the tanker are given in Table I. The body plan is shown in Fig. 2. The set-up of the mooring is given in Fig. 3.

Computations

For the computations the tanker was exposed to the wave spectra given in Fig. 11. The motions have been calculated by means of frequency domain computations. For the spectral density of the wave drift forces $S_{X_1}(\mu=0)$ use is made of the the computed transfer function of the wave drift forces as given in Table V. The mean drift damping has been computed by means of the data given in Fig. 12.

The still water damping has been experimentally determined by means of a decay test and amounts to B_{11} = 24.4 tf.s/m. The results of the computations on the spectral densities of the wave drift forces and the mean wave drift damping are given in Table VI. The results of the computations with and without wave drift damping in terms of the root-mean square values of the low frequency surge motion are given in Table VI and in Fig. 13.

Model Tests

To validate the results of the computations model tests in irregular waves were carried out in the Seakeeping Laboratory of MARIN. The basin measures 60x20x2.5 m. The model scale was 82.5. All data presented were scaled to full scale according to Froude's law of similitude. The loaded 200 kDWT tanker was used for the tests. The test set-up is given in Fig. 3. The total spring in surge direction corresponded to c_{11} = 13.6 tf/m. During the tests the surge motion was measured in the COG by means of an optical tracking device.

Prior to the installation of the test set-up the wave spectra were adjusted with the wave probe at the projected location of the COG of the tanker. The wave spectra are presented in Fig. 11. Each sea state was prepared for a test duration of 2.5 hours for full scale time.

The tests were performed in wave spectra No. 1, 2, 5 and 6. The measured wave spectra and the associated spectra of the groups are presented in the Figs. 11 and 14. The results of the tests in terms of the root-mean square values of the low frequency motions are given in Table VI and in Fig. 13. The result of a decay test in still water is given in Fig. 15.

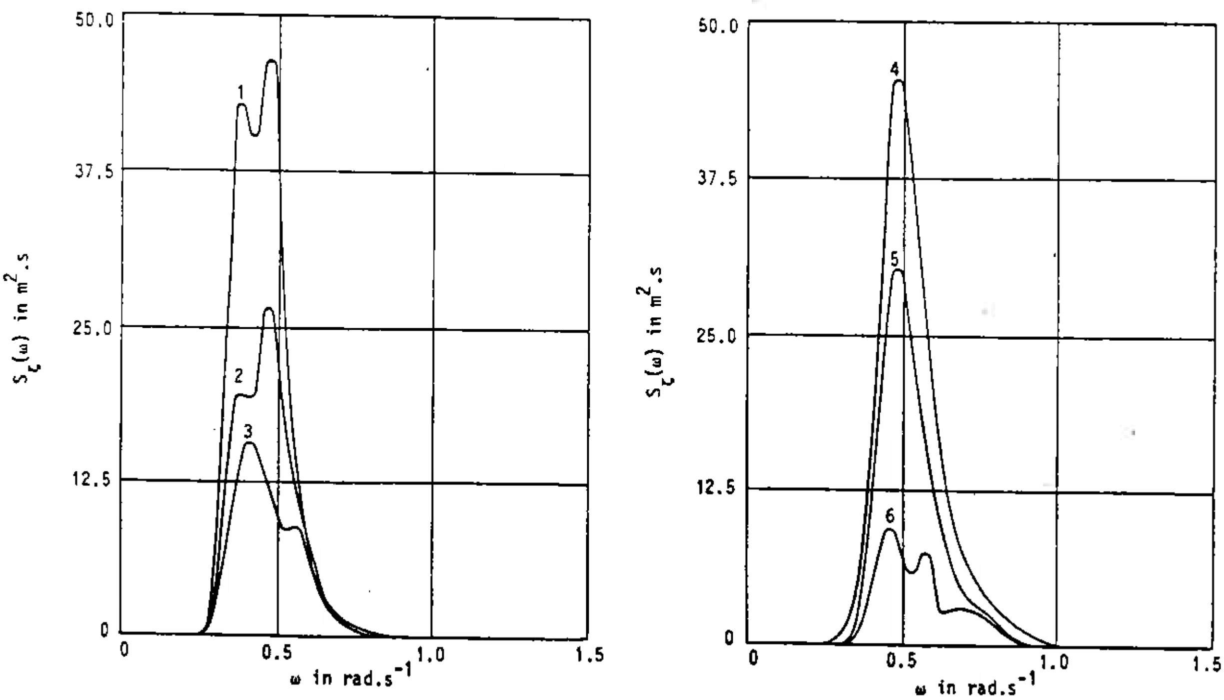

Fig. 11 The spectral densities of the wave spectra

Table V: Quadratic transfer function of wave drift force of the 200 kDWT tanker in 206 m water depth

$T(\omega_i, \omega_i)$

ω_1 \ ω_2	0.08	0.16	0.24	0.32	0.40	0.48	0.56	0.64	0.72	0.80	0.88	0.96	1.04
0.08	0												
0.16		0											
0.24			0.1										
0.32				0.8									
0.40					3.5								
0.48						8.7							
0.56							12.9						
0.64								11.9					
0.72									8.6				
0.80										9.2			
0.88	302 facets - 74 waterline elements										8.7		
0.96	Frequencies in rad/s											8.7	
1.04	Water depth 206 m												8.8

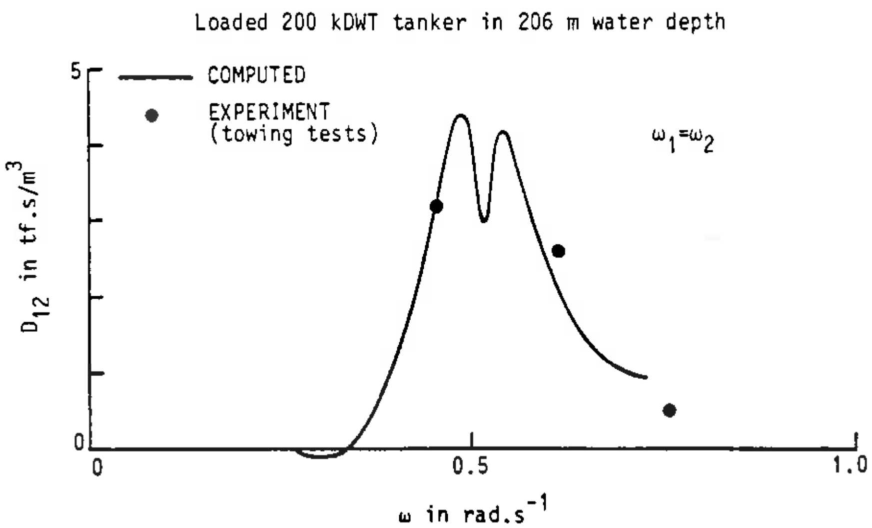

Fig. 12 Quadratic transfer function of the wave drift damping coefficient for the 200 kDWT tanker in head waves (earth-bound wave frequencies) [2]

Table VI: Results of computation and model tests

Wave spectrum					Calculated data according to frequency domain			
No.	$\bar{\zeta}_{w1/3}$ (m)	$\bar{\bar{T}}_1$ (s)	c_{11} (tf/m)	Measured σ_{x_1} (m)	With wave drift damping σ_{x_1} (m)	Without wave drift damping σ_{x_1} (m)	Spectral density drift forces S_{X_1} (μ=0) (tf^2.s)	Mean wave drift damping B_1 (tf.s/m)
1	12.50	14.1	13.6	13.50	14.5	25.1	132,625	48.75
2	9.40	13.7	13.6	10.38	10.5	15.4	49,835	27.60
3	7.50	13.6	13.6	-	7.2	9.0	17,161	14.14
4	12.50	12.0	13.6	-	18.4	32.8	227,741	53.7
5	9.80	12.0	13.6	12.80	13.4	20.9	92,087	34.4
6	6.10	11.4	13.6	6.74	5.5	6.6	9,321	10.6

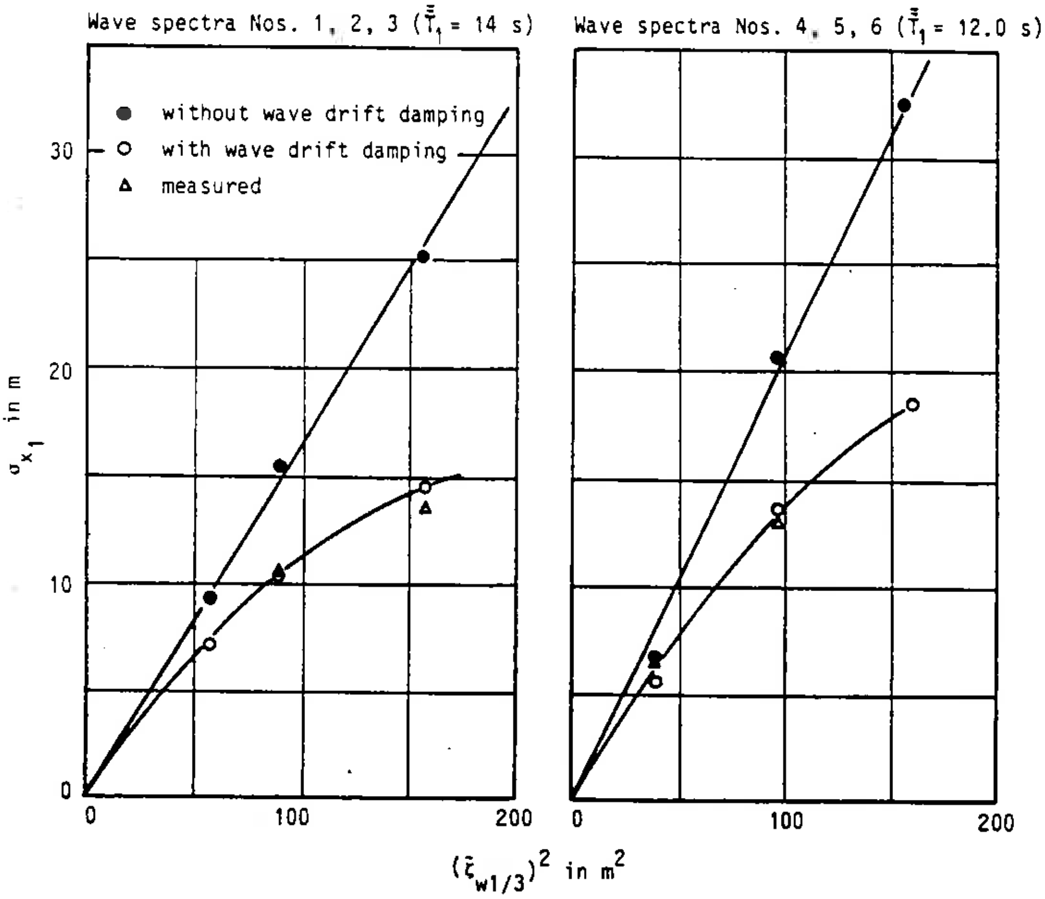

Fig. 13 Root-mean square values of the low frequency surge motion versus significant wave height squared

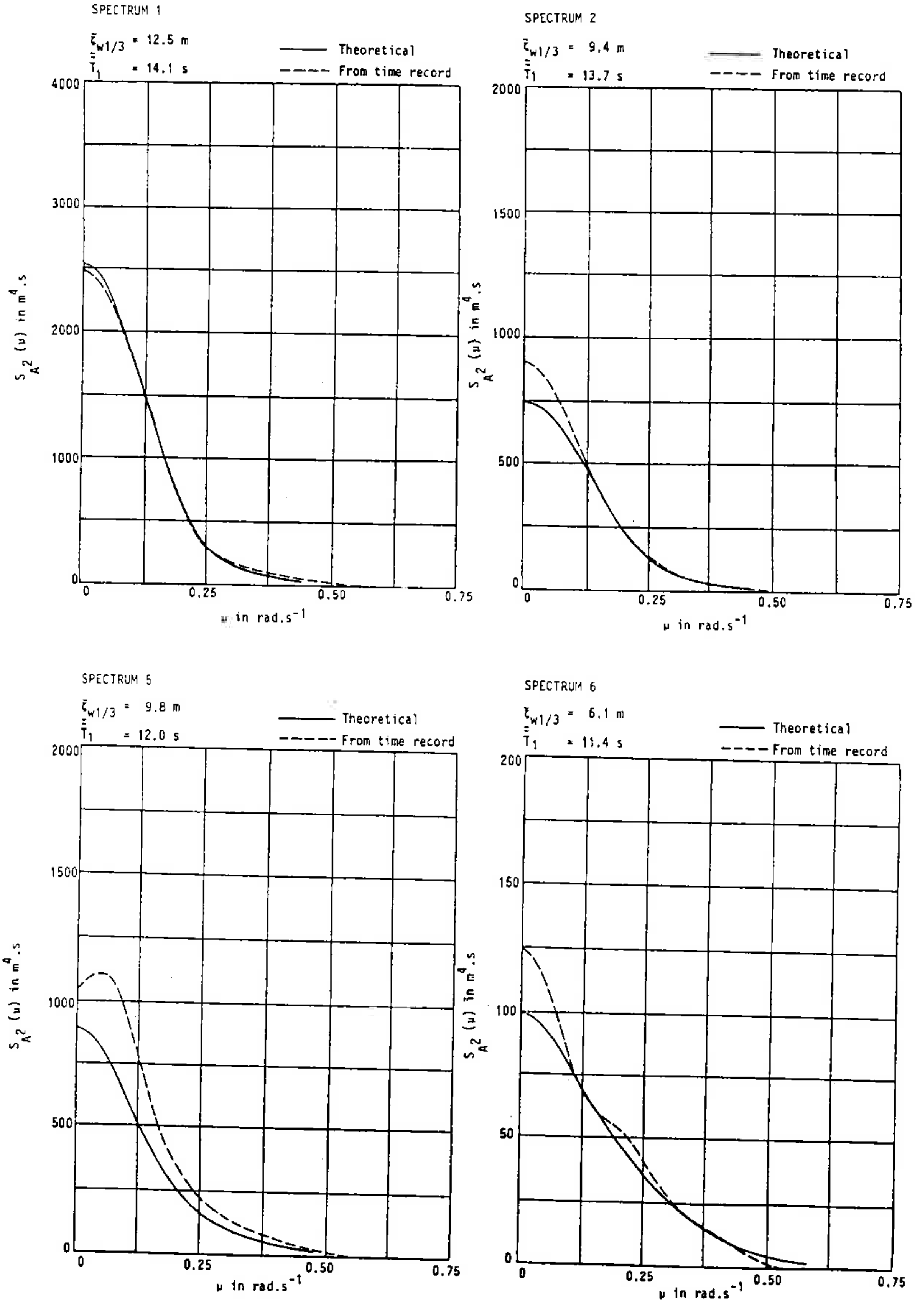

Fig. 14 The spectra of the wave groups

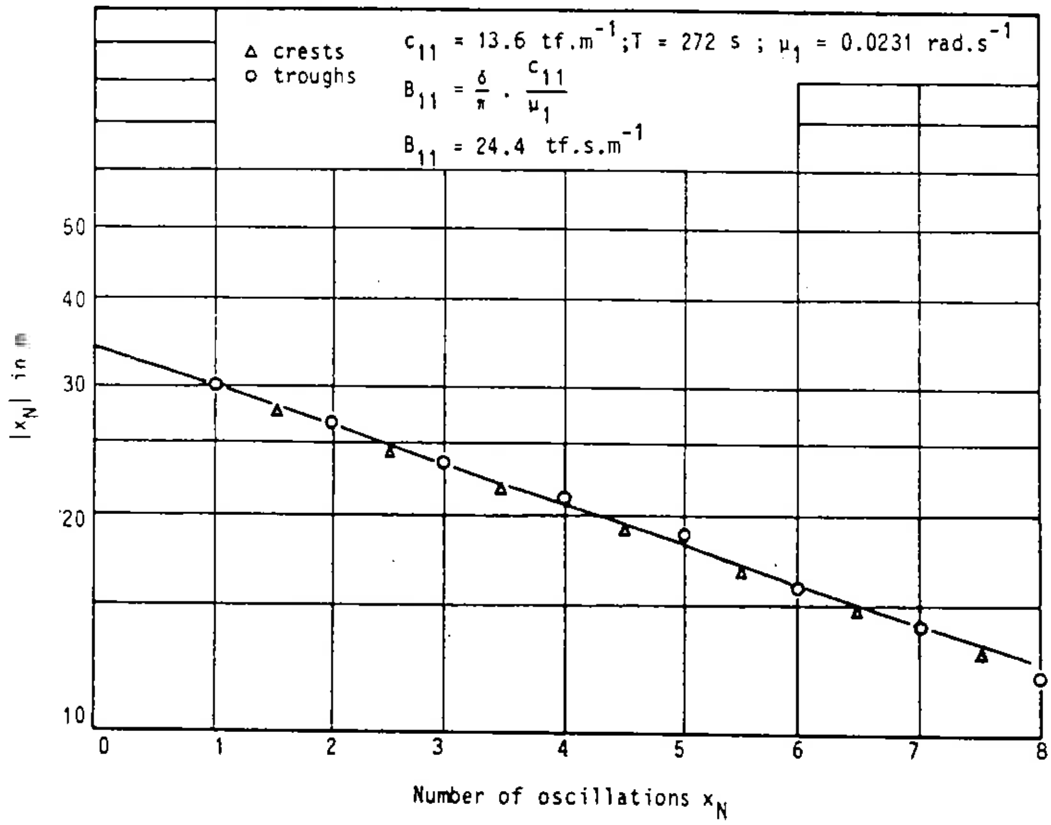

Fig. 15 The measured still water damping

Discussion Of Results

The viscous damping as measured for the tanker with the spring constant c_{11} = 13.6 tf/m amounts to B_{11} = 24.5 tf.s/m. The damping was expected to be the same or lower than found for the spring stiffness c_{11} = 17.7 tf/m.

With regard to the higher sea states, see Table VI, the wave drift damping is the dominant contribution to the total damping. Significant deviations in motion will occur if the wave drift damping will not be taken into account.

A good agreement is found between the measured and computed motions of the tanker in the irregular waves if the wave drift damping is taken into account. Some difference in the standard deviations can occur and are caused by the statistical convergence of the test duration of 2.5 hours full scale. The frequency domain results refer to the statistical convergence for a duration of infinity. The statistical convergence of the standard deviation of the motions as function of test or simulation duration is described by Pinkster and Wichers [6].

Conclusions

- The contribution of the oscillating part of the wave drift damping will be negligibly small. The constant part of the wave drift damping, especially in high seas combined with the viscous damping, will determine the low frequency motion of the tanker.
- The computational code for the prediction of the low frequency motions of a tanker exposed to regular wave groups and irregular head waves gives satisfactory results.

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