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Wave-Current Interaction Effects on Moored Tankers in High Seas

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ABSTRACT

A tanker moored in irregular head waves in combination with or without current performs large amplitude low frequency surge motions. Due to low frequency motions in combination with or without the current field the second order wave drift forces will be a function of the tanker velocities.

By means of 3-D potential theory the quadratic transfer function of the wave drift force for zero speed and low forward speed were computed. By means of the gradient-method the transfer function of the constant speed dependent wave drift force and the wave drift damping coefficient were derived. The transfer function of the wave drift damping coefficient is compared with the results of model tests.

By means of the obtained results time-domain computations on the low frequency surge motions in irregular waves with and without current were carried out. The results of the computed low frequency motions were compared with the results of model tests. The time-domain computations are based on the wave train registrations as were adjusted for the model tests.

Prior to solving the equation of motion the mean wave drift damping coefficient and the registration of the wave drift force with and without current were computed. The results of the computed wave drift force registration with and without current in terms of spectral densities were compared with the results of model tests.

INTRODUCTION

The motions of a moored tanker in irregular head waves consist of small amplitude high (= wave) frequency surge, heave and pitch motions and large amplitude low frequency surge motions. The frequencies of the low frequency surge motions are concentrated around the natural frequency of the system, see Fig. 1.

References and illustrations at end of paper.

To study the motions use has been made of two different systems of axes as indicated in Fig. 2; the system of axes Ox(1)x(3) is fixed in space, with the Ox(1)- and the Ox(3)-axis coinciding with the ship-fixed system of axes Gx1x3, at rest.

We shall assume that the surge, heave and pitch motions can be decoupled into the following form:

$$x_1 = \varepsilon x_1^{(1)}(t/\eta,t) + \varepsilon^2\left(x_{1lf}^{(2)}(t) + x_{1hf}^{(2)}(\tfrac{2t}{\eta},t)\right)$$

$$x_3 = \varepsilon x_3^{(1)}(t/\eta,t) + \varepsilon^2\left(x_{3lf}^{(2)}(t) + x_{3hf}^{(2)}(\tfrac{2t}{\eta},t)\right)$$

$$x_5 = \varepsilon x_5^{(1)}(t/\eta,t) + \varepsilon^2\left(x_{5lf}^{(2)}(t) + x_{5hf}^{(2)}(\tfrac{2t}{\eta},t)\right) \quad . . (1)$$

with ε and η being small parameters, viz.:
- ε relates to the wave steepness;
- η considers the ratio between the two time scales of the motions: the μ frequency range of the natural frequency of the system and the ω frequency range of the wave spectrum frequencies.

and further;
- $x_1^{(1)}$, $x_3^{(1)}$ and $x_5^{(1)}$ relate to the wave frequency surge, heave and pitch motions;
- $x_{1lf}^{(2)}$, $x_{3lf}^{(2)}$ and $x_{5lf}^{(2)}$ stands for the large amplitude low frequency second order surge, heave and pitch motions;
- $x_{1hf}^{(2)}$, $x_{3hf}^{(2)}$ and $x_{5hf}^{(2)}$ represent the second order motions of which the frequency range is twice the wave frequency range.

Of the second order motions only the low frequency part will be considered and will be denoted as $\underline{x}^{(2)}$.

As an introduction to the problem of the velocity dependency of the hydrodynamic forces a simplified mathematical model of a linearly moored tanker will be considered:
- the tanker will be exposed to a regular head wave with frequency ω;
- the linear spring of the mooring system will be c_{11};
- a low frequency oscillating external force acting in surge direction will be applied to the tanker:

$$\tilde{X}_1(t) = X_{1a} \cos\mu_1 t \quad \ldots \ldots \ldots \ldots \ldots (2)$$

in which μ_1 = natural frequency of the system in surge direction.

The total wave exciting forces as present in regular head waves consist of the following parts:

$$X_k(t) = X_k^{(1)}(t) + X_k^{(2)} \quad \text{for } k = 1, 3, 5 \quad \ldots (3)$$

where $X_k^{(1)}(t)$ is the first order wave exciting force and $X_k^{(2)}$ represents the mean wave drift force. Due to these wave forces the result will be that the tanker performing high frequency motions around a mean displacement.

It is assumed that the natural frequency of the moored tanker in surge direction is very low. At low frequencies the damping is small. Due to the external force $\tilde{X}_1$ the result will be that in surge direction large amplitude low frequency motions combined with the high frequency motions will occur.

The total fluid damping in surge direction will be caused by the combined high and low frequency motions. Since for the low frequencies negligibly small damping due to wave radiation exists in surge direction ($\omega < 0.08$ rad/s; see ref. [1]), the fluid damping force is assumed to be of viscous origin. Because the origin of the damping mechanisms for the first and second order surge motions are completely different (wave radiation versus viscosity) it is assumed that the damping forces will not interfere with each other. To this end we assume that the wave radiation is excited through the first order motions, while the viscous damping forces can be generated by the first and second order motions.

Based on the aforementioned assumptions the equations of motion can be written as follows:

$$\left(M_{11} + a_{11}(\mu_1)\right)\ddot{x}_1^{(2)} + B_{11}(\mu_1)(\dot{x}_1^{(2)} + \dot{x}_1^{(1)}) +$$

$$+ B_{112}(\mu_1)(\dot{x}_1^{(2)} + \dot{x}_1^{(1)}) \left| \dot{x}_1^{(2)} + \dot{x}_1^{(1)} \right| + c_{11}x_1^{(2)} +$$

$$+ \sum_{j=1,3,5} \left\{ \left(M_{1j} + a_{1j}(\omega)\right)\ddot{x}_j^{(1)} + b_{1j}(\omega)\dot{x}_j^{(1)} + \right.$$

$$\left. + c_{1j}x_j^{(1)} \right\} = X_1(t) + \tilde{X}_1(t)$$

and

$$\sum_{j=1,3,5} \left\{ \left(M_{kj} + a_{kj}(\omega)\right)\ddot{x}_j + b_{kj}(\omega)\dot{x}_j + c_{kj}x_j \right\}$$

$$= X_k(t) \quad \text{for } k = 3, 5 \quad \ldots \ldots \ldots \ldots \ldots (4)$$

in which:

M_{kj} = inertia matrix of the vessel
$a_{kj}(\omega)$ = added mass matrix for wave frequency ω
$b_{kj}(\omega)$ = damping matrix for wave frequency ω
c_{kj} = matrix of restoring coefficients
$a_{11}(\mu_1)$ = added mass coefficient in surge direction at frequency μ_1
$B_{11}(\mu_1)$ = linear viscous damping coefficient in surge direction at frequency μ_1
$B_{112}(\mu_1)$ = quadratic viscous damping coefficient in surge direction at frequency μ_1
μ_1 = natural frequency of the system in surge direction
x_j = $x_j^{(1)} + x_j^{(2)}$

Oscillating at the high frequencies and simultaneously performing the low frequency large amplitude oscillations the hydrodynamic reaction forces will be affected by the slowly varying speed. Further due to the low frequency large amplitude oscillations through the regular wave field the wave forces will be affected by both the displacement and the speed. The actual high frequency hydrodynamic reaction coefficients and the wave forces should be written as follows:

$$a_{kj}(\omega, \dot{x}_1^{(2)})$$

$$b_{kj}(\omega, \dot{x}_1^{(2)})$$

$$X_k^{(1)}(x_1^{(2)}, \dot{x}_1^{(2)}, t)$$

$$X_k^{(2)}(\underline{x}^{(1)}, x_1^{(2)}, \dot{x}_1^{(2)}) \quad \text{for } j = 1, 3, 5 \quad \ldots (5)$$

By applying a Taylor series to the wave drift force for first order variations in surge direction we will obtain:

$$X_1^{(2)}(\underline{x}^{(1)}, x_1^{(2)}, \dot{x}_1^{(2)}) = X_1^{(2)}(\underline{x}^{(1)}, 0, 0) +$$

$$+ \frac{\partial X_1^{(2)}(\underline{x}^{(1)}, 0, 0)}{\partial x_1^{(2)}} \cdot x_1^{(2)} + \frac{\partial X_1^{(2)}(\underline{x}^{(2)}, 0, 0)}{\partial \dot{x}_1^{(2)}} \cdot \dot{x}_1^{(2)} \quad \ldots (6)$$

Of the second order wave drift force in a regular wave, as is indicated in eq.(6), the first term is the mean wave drift force and will be a constant.

Since the mean wave drift force is independent of the position of the tanker in the regular waves the derivative to the displacement will be zero.

After inspection of the terms of eq. (4) the equation of the low frequency motion in surge direction can reduced as follows:

$$\left(M + a_{11}(\mu_1)\right)\ddot{x}_1^{(2)} = -B_{11}(\mu_1)\dot{x}_1^{(2)} - B_{112}(\mu_1)\dot{x}_1^{(2)} \left| \dot{x}_1^{(2)} \right| +$$

$$- B_1(\omega)\dot{x}_1^{(2)} - c_{11}x_1^{(2)} + X_1^{(2)}(\underline{x}_1^{(1)}, 0, 0) + \tilde{X}_1(t) \quad . (7)$$

in which: $B_1(\omega)$ = wave damping coefficient

$$= - \frac{\partial X_1^{(2)}(\underline{x}_1^{(1)}, 0, 0)}{\partial \dot{x}^{(2)}}$$

In the right hand side of eq. (7) three low frequency damping coefficients can be recognized. The first two terms are assumed to be of viscous nature while the last term relates to the low frequency velocity on the mean wave drift force. In order to analyze and verify the separate terms model tests were carried out. The following experiments were performed:

1. Motion decay tests:
   - in still water;
   - in regular head waves with various wave heights and periods.

2. Towing tests at low vessel speed:
   - in still water;
   - in regular head waves with various wave heights and periods.

The experiments and the results have been reported in ref. [2]. The tanker concerns the loaded 200 kTDW tanker. The particulars are given in Table 1, while the body plan is shown in Fig. 3. The motion decay tests were carried out in 82.5 m and the towing tests in 206 m water depth. The results of the experimentally determined transfer function of the wave drift damping coefficient are given in Fig. 4.

The trend of the experimentally determined transfer function is supported by the results of the experiments reported in ref. [3] and [4].

From the experimental findings in ref. [2] it was concluded that the expansions used in eq. (7) hold true for the added resistance gradient for low forward speed and that the gradient corresponds to the wave drift damping coefficient:

$$\frac{B_1(\omega)}{\zeta_a^2} = - \left. \frac{\partial X_1^{(2)}(\dot{x}_1^{(2)},\underline{x}^{(1)},\omega)}{\zeta_a^2 \partial \dot{x}_1^{(2)}} \right|_{\dot{x}_1^{(2)}=0} = - \left. \frac{\partial X_1^{(2)}(U,\underline{x}^{(1)},\omega)}{\zeta_a^2\, \partial U} \right|_{U=0} \quad \ldots\ldots\ldots (8)$$

and if the tanker performs low frequency oscillations in the wave field the total transfer function of the wave drift force in a regular wave with frequency ω can be written as follows:

$$\frac{X_1^{(2)}(\dot{x}_1^{(2)},\omega)}{\zeta_a^2} = \frac{X_1^{(2)}(0,\omega)}{\zeta_a^2} - \frac{B_1(\omega).\dot{x}_1^{(2)}}{\zeta_a^2} \quad \ldots\ldots (9)$$

From the experiments it was found that the wave drift force approximately linearly increased for low forward speeds. Based on the gradient the following method can be used to approximate the quadratic transfer function of the wave drift force as function of low vessel speed U in a regular wave with frequency ω:

$$\frac{X_1^{(2)}(U,\omega)}{\zeta_a^2} = \frac{X_1^{(2)}(0,\omega)}{\zeta_a^2} - \frac{B_1(\omega).U}{\zeta_a^2} \quad \ldots\ldots\ldots (10)$$

and if the tanker performs low frequency oscillations in the wave field superimposed on the steady towing speed U the total transfer function of the wave drift force can be approximated as follows:

$$\frac{X_1^{(2)}(U+\dot{x}_1^{(2)},\omega)}{\zeta_a^2} = \frac{X_1^{(2)}(0,\omega)}{\zeta_a^2} - \frac{B_1(\omega).(U+\dot{x}_1^{(2)})}{\zeta_a^2} \quad \ldots (11)$$

These procedures will further be referred to as the gradient-method. For all the towed conditions the wave frequencies are referred to an earth-bound system of coordinates.

COMPUTATION ON THE LOW VELOCITY DEPENDENT WAVE DRIFT FORCES

Theory

The transfer function of the wave drift force for zero speed in regular waves can be computed by the direct pressure integration method as given in ref. [5]. For zero speed the input of the direct pressure integration method may be based on the output of the diffraction program as reported in ref. [6]. The diffraction program treats the ship motions for the zero speed case without any geometrical simplification of the under water hull. The program is based on the solution of integral equations, where the potential function is written as a source distribution along the hull.

In order to determine the gradient or the wave drift damping coefficient at zero speed computations for small values of forward speed are necessary. For the computation of the velocity dependent wave drift forces the diffraction program was adapted for the speed effects.

The adaptation of the diffraction program is restricted to regular waves in deep water and is based on small values of forward speed.

For the introduction of the forward speed U the total potential function can be split up in a steady and a non-steady part in a well-known way:

$$\Phi(\underline{x},t) = -U.x_1 + \bar{\Phi}(\underline{x},U) + \tilde{\Phi}(\underline{x},t,U) \quad \ldots\ldots (12)$$

in which $\underline{x} = \underline{x}^{(1)}$

The steady part does not contribute to the non-steady part directly. It plays a role in the free surface condition. Because of the very low Froude numbers ($Fn = U/\sqrt{g.L} << .1$) the effect of the free-surface is not taken into account and the contribution of $\bar{\phi}(\underline{x},U)$ is totally neglected. The time dependent oscillatory potential $\tilde{\Phi}(\underline{x},t,U)$ will be written as a source distribution along the hull and the water line and will be expanded with respect to small values of U. By solving the potential the part linear with speed will lead to the speed effects in the ship motions. By applying the direct integration method the transfer function of the wave drift force for zero and small values of forward speed U can be computed.

Plotting the values of the quadratic transfer function on base of the small values of forward speed, the wave drift damping coefficient can be determined.

For the theory on the computations reference is made to [7], [8] and [9].

Results of computations

The computations of the quadratic transfer function of the second order wave drift force and the wave drift damping coefficient have been applied to the loaded 200 kTDW tanker sailing with small values of forward speed in deep water. Based on the transfer function for zero, 1 kn and 2 kn forward speed the gradients at zero speed have been determined in order to obtain the transfer function of the wave drift damping coefficient. The results derived from the computations together with the experimental data have been plotted in Fig. 5.

Comparing the results of the model tests and the computations it can be concluded that a reasonable approximation can be achieved by means of the potential theory with low forward speed. A peculiar deviation occurs at wave frequency $\omega = 0.523$ rad/s. The results clearly indicate that the damping caused by the waves is dominated by the velocity potential. Similar results were found in ref. [10]. Viscous drag due to the orbital motions of the fluid particles in combination with the low frequency tanker motions may be neglected as suggested in ref. [11] and ref. [12].

THE LOW FREQUENCY COMPONENTS OF THE WAVE DRIFT FORCES AND THE WAVE DRIFT DAMPING COEFFICIENTS

Wave drift force in irregular waves for zero speed

The foregoing sections dealt with the transfer functions of the wave drift forces and the wave drift damping coefficient for regular waves only. In irregular waves, however, for both the wave drift forces and the wave drift damping coefficients mean and low frequency components may occur. The frequencies of the low frequency components are associated with the frequencies of the wave groups.

It is assumed that both the transfer function of the wave drift force for zero speed and the wave drift damping coefficients as obtained in regular waves are known. Based on these data approximations will be applied to compute the low frequency components of the wave drift forces of the total wave drift forces with and without current. These approximations are allowed for relatively deep water, mono-hull type vessels and for very low values of the natural frequencies of the system.

In order to arrive at the method of the approximations to compute the low frequency components for the total wave drift force first the derivation will be given for the wave drift forces for zero speed as described in [5].

The behaviour of the drift forces in waves can be elucidated by first looking at the general expression for the drift forces in a wave train consisting of two regular sinusoidal waves with frequencies ω_1 and ω_2 and amplitudes ζ_1 and ζ_2. The wave elevation is written as:

$$\zeta(t) = \sum_{i=1}^{2} \zeta_i \sin(\omega_i t + \varepsilon_i)$$

$$= \zeta_1 \sin(\omega_1 t + \varepsilon_1) + \zeta_2 \sin(\omega_2 t + \varepsilon_2) \quad \ldots (13)$$

For small differences between ω_1 and ω_2 the wave train will be denoted by a regular wave group. This type of wave train is characterized by a periodic variation of the wave envelope. The frequency associated with the envelope is equal to $\Delta\omega = \omega_1 - \omega_2$ being the difference frequency of the regular wave components. We will write the wave elevation in amplitude modulated form:

$$\zeta(t) = A(t) \sin(\bar{\omega} t + \bar{\varepsilon}) \quad \ldots \ldots \ldots \ldots (14)$$

in which: $\bar{\omega} = (\omega_1 + \omega_2)/2$

$\bar{\varepsilon} = (\varepsilon_1 + \varepsilon_2)/2$

It can be shown that the envelope becomes:

$$A(t) = \left[\sum_{i=1}^{2} \sum_{j=1}^{2} \zeta_i \zeta_j \cos\left((\omega_i - \omega_j)t + (\varepsilon_i - \varepsilon_j)\right)\right]^{\frac{1}{2}} \; . \; (15)$$

The square of the envelope is:

$$A^2(t) = \sum_{i=1}^{2} \sum_{j=1}^{2} \zeta_i \zeta_j \cos\left((\omega_i - \omega_j)t + (\varepsilon_i - \varepsilon_j)\right) \quad \ldots (16)$$

A quantity which is a quadratic function of the wave amplitudes, in this case the wave drift force, will be as follows:

$$X_1^{(2)}(t) = \sum_{i=1}^{2} \sum_{j=1}^{2} \zeta_i \zeta_j P_{ij} \cos\left((\omega_i - \omega_j)t + (\varepsilon_i - \varepsilon_j)\right) +$$

$$+ \sum_{i=1}^{2} \sum_{j=1}^{2} \zeta_i \zeta_j Q_{ij} \sin\left((\omega_i - \omega_j)t + (\varepsilon_i - \varepsilon_j)\right) \quad \ldots \ldots \ldots \ldots (17)$$

in which P_{ij} and Q_{ij} are quadratic transfer functions dependent on two frequencies ω_i and ω_j. Generally P_{ij} and Q_{ij} are computed in such a way that the following relations exist:

$$P_{ij} = P_{ji}$$

$$Q_{ij} = - Q_{ji}$$

P_{ij} is that part of the quadratic transfer function which expresses the component of the drift force which is in-phase with the square of the wave envelope and Q_{ij} expresses the quadrature part of the drift forces. For the regular wave group the wave drift force is:

$$X_1^{(2)}(t) = \zeta_1^2 P_{11} + \zeta_2^2 P_{22} + \zeta_1\zeta_2(P_{12}+P_{21}) \cdot \cos\big((\omega_1-\omega_2)t + (\varepsilon_1-\varepsilon_2)\big) + \zeta_1\zeta_2(Q_{12}-Q_{21})\sin\big((\omega_1-\omega_2)t + (\varepsilon_1-\varepsilon_2)\big) \cdot \quad (18)$$

The formulation shows that the drift force contains several components. The first two are constant parts corresponding to the mean drift force in each of the regular wave components separately. The third and fourth parts are low frequency varying components which arise through the combined presence of the two regular wave compontents in the wave group.

The quadratic transfer function of the wave drift force in terms of amplitudes and phase angles are defined as follows:

$$T_{ij} = T(\omega_i,\omega_j) = \big(P^2(\omega_i,\omega_j) + Q^2(\omega_i,\omega_j)\big)^{\frac{1}{2}} \quad \ldots (19)$$

= quadratic transfer function of the amplitude of the wave drift force.

$$\varepsilon_{ij} = \arctan - \frac{Q(\omega_i,\omega_j)}{P(\omega_i,\omega_j)} \quad \ldots\ldots\ldots\ldots (20)$$

= phase angle between the low frequency part of the second order force relative to the low frequency part of the square of the wave elevation.

Using this definition of the quadratic transfer function the wave drift force for the regular wave group can be written as follows:

$$X_1^{(2)}(t) = \sum_{i=1}^{2}\sum_{j=1}^{2} \zeta_i\zeta_j T_{ij}\cos\big((\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)+\varepsilon_{ij}\big) \quad \ldots\ldots\ldots\ldots (21)$$

In irregular waves the wave drift force is:

$$X_1^{(2)}(t) = \sum_{i=1}^{N}\sum_{j=1}^{N} \zeta_i\zeta_j T_{ij}\cos\big((\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)+\varepsilon_{ij}\big) \quad \ldots\ldots\ldots\ldots (22)$$

The quadratic transfer function P_{ij} and Q_{ij} for zero speed can be computed by means of the direct pressure integration method [9].

<u>Approximation of the low frequency components</u>

At present the computations of the wave drift forces with low forward speed have been developed for regular waves; the values for $P(U,\omega_i,\omega_i)$ can be computed only. Therefore for all the computations the following assumption is made:

$$P(\omega_i,\omega_i) = \frac{X_1^{(2)}(\omega_i)}{\zeta_a^2}$$

and

$$Q(\omega_i,\omega_i) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

In order to estimate the unknown low frequency varying components of the drift forces an approximation will be made. The approximation is possible if the water depth is large and the natural frequency of the system is very low. In ref. [5] it is shown that the influence of the second order potential in deep water is negligibly small on the low frequency parts. Neglecting the effect of the second order potential the low frequency varying part arises through the combined presence of two regular wave components. The frequency difference of importance will be $\omega_i-\omega_j = \mu$, in which μ is very small. If μ is small then $Q(\omega_i,\omega_j) \approx 0$. Neglecting the quadrature part of the transfer function the low frequency component can be estimated as proposed in ref. [13]:

$$P(\omega_i,\omega_j) = \frac{\big(P(\omega_i,\omega_i) + P(\omega_j,\omega_j)\big)}{2} \quad \ldots\ldots (24)$$

Because the frequency difference is assumed to be small eq. (24) will approximately correspond to:

$$P(\omega_i,\omega_j) = P\Big(\frac{\omega_i+\omega_j}{2},\frac{\omega_i+\omega_j}{2}\Big)$$

or

$$T(\omega_i,\omega_j) = P\Big(\frac{(\omega_i+\omega_j)}{2},\frac{(\omega_i+\omega_j)}{2}\Big) \quad \ldots\ldots\ldots (25)$$

This approximation will be used for all computations.

<u>Total wave drift force in irregular waves without current</u>

Following the gradient-method the total wave drift force in a regular wave group can be written as follows:

$$X_{1t}^{(2)}(t) = X_1^{(2)}(t) + \frac{\partial X_1^{(2)}(t)}{\partial \dot{x}_1}\dot{x}_1 = \zeta_1^2 T_{11} + \zeta_2^2 T_{22} + 2\zeta_1\zeta_2 T_{12}\cos(\Delta\omega_{12}t+\Delta\varepsilon_{12}) + \zeta_1^2 D_{11}\dot{x}_1 + \zeta_2^2 D_{22}\dot{x}_1 + 2\zeta_1\zeta_2 D_{12}\cos(\Delta\omega_{12}t + \Delta\varepsilon_{12})\dot{x}_1 \quad \ldots (26)$$

in which: $\dot{x}_1 = \dot{x}_1^{(2)}$

$$D_{11} = \frac{\partial T_{11}}{\partial \dot{x}_1}$$

$$D_{22} = \frac{\partial T_{22}}{\partial \dot{x}_1}$$

$$D_{12} = \frac{\partial T_{12}}{\partial \dot{x}_1}$$

The total wave drift force can be split in a wave drift force and a wave drift damping part. The wave drift damping force contains several components. The coefficient of the first two terms corresponds to the mean wave drift damping in each of the regular wave components seperately. The third term stands for the low frequency varying part of the wave drift damping force. As mentioned for the derivative of $T(\omega_i,\omega_j)$ to the low frequency velocity no data exists. To estimate the oscillating part of the damping the same procedure is proposed as applied to the oscillating part of the wave drift forces:

$$D_{ij} = \frac{\partial T(\omega_i,\omega_j)}{\partial \dot{x}_1} = \frac{\partial T\left(\frac{\omega_i+\omega_j}{2},\frac{\omega_i+\omega_j}{2}\right)}{\partial \dot{x}_1} \quad \ldots\ldots (27)$$

The total wave drift force in irregular waves without current will be:

$$X_{1t}^{(2)} = \sum_{i=1}^{N}\sum_{j=1}^{N} \zeta_i\zeta_j T_{ij}\cos\left((\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)+\varepsilon_{ij}\right) +$$

$$+ \sum_{i=1}^{N}\sum_{j=1}^{N} \zeta_i\zeta_j D_{ij}\dot{x}_1 \quad \ldots\ldots\ldots\ldots (28)$$

Stability of the solution and contribution of the oscillating part of the wave drift damping

The effect of the damping will play an important role for the condition that the natural frequency of the moored vessel will correspond to the frequency of the wave group:

$$\Delta\omega_{12} = \mu = \sqrt{\frac{c_{11}}{M + a_{11}(\mu)}}$$

Assuming a linear viscous damping the equation of low frequency motion can be written as follows:

$$\left(M + a_{11}(\mu)\right)\ddot{x}_1 + \left(B_{11} - \zeta_1^2 D_{11} - \zeta_2^2 D_{22} - \right.$$

$$\left. - 2\zeta_1\zeta_2 D_{12}\cos(\mu t + \varepsilon_{12})\right)\dot{x}_1 + c_{11}x_1 =$$

$$= \zeta_1^2 T_{11} + \zeta_2^2 T_{22} + 2\zeta_1\zeta_2 T_{12}\cos(\mu t+\varepsilon_{12}) \quad \ldots (29)$$

The damping term contains linear coefficients and a low frequency oscillating coefficient. Due to the low frequency oscillating coefficient the value of the damping coefficient will be positive or negative. Since a negative damping in the equation of motion can cause an unstable solution, attention has to be paid to the magnitude of the oscillating damping coefficient with regard to the linear damping coefficients. Moreover, attention will be paid to the contribution of the damping force with the oscillating coefficient to the motions.

In order to judge the influence of the low frequency oscillating coefficient on the stability of the solution the equation of motion is simplified as follows:

$$m\ddot{x}_1 + f(t)\dot{x}_1 + cx_1 = a.\cos\mu t \quad \ldots\ldots\ldots (30)$$

in which: $f(t) = b_1 + b_2\cos\mu t$
$a.\cos\mu t$ = oscillating part of the wave drift force

Starting from the equation of motion for the tanker

$$m\ddot{x}_1 + f(t)\dot{x}_1 + cx_1 = 0 \quad \ldots\ldots\ldots\ldots (31)$$

and multiplying this equation by $\dot{x}_1$, we obtain:

$$m\ddot{x}_1\dot{x}_1 + f(t)\dot{x}_1^2 + cx_1\dot{x}_1 = 0 \quad \ldots\ldots\ldots\ldots (32)$$

or in terms of energy:

$$\frac{d}{dt}\left(\tfrac{1}{2}m\dot{x}_1^2 + \tfrac{1}{2}cx^2\right) = - f(t)\dot{x}_1^2 \quad \ldots\ldots\ldots\ldots (33)$$

which means that the decrease of the total energy corresponds to $f(t).\dot{x}_1^2$. The system is called instable if the term $f(t)$ is negative. In order to judge the stability the sign of $f(t)$ will be studied.

The damping function $f(t)$ consists of still water damping and the derivatives of wave drift force components to the low frequency velocity.

Because the natural frequency of the moored vessel is usually very low, the frequency difference of importance will be $\omega_i-\omega_j = \mu$ and $\mu \ll 1$.

Caused by the small value of the frequency difference the magnitudes of the damping coefficients will have approximately the same value:

$$\frac{\partial T(\omega_i,\omega_i)}{\partial \dot{x}_1} \approx \frac{\partial T(\omega_j,\omega_j)}{\partial \dot{x}_1} \approx \frac{\partial T(\omega_i,\omega_j)}{\partial \dot{x}_1} \quad \ldots\ldots (34)$$

and since:

$$\zeta_i^2 + \zeta_j^2 > 2\zeta_i\zeta_j \quad \ldots \ldots (35)$$

and because the still water damping has to be added to the linear parts of the wave damping coefficients it can be concluded that the low frequency oscillating damping coefficient will be smaller than the linear damping coefficient. Therefore the sign of f(t) will be positive for all values of t.

Besides the stability of the solution also attention will be paid to the contribution of the oscillating damping to the motion. For the equation of motion

$$m\ddot{x} + (b_1 + b_2\cos\mu t)\dot{x}_1 + cx_1 = a\cos\mu t \quad \ldots \ldots (36)$$

the following solution has been assumed:

$$x_1 = p_0 + \sum_{n=1}^{\infty} \left(p_n\cos(n\mu t) + q_n\sin(n\mu t)\right) \quad \ldots \ldots (37)$$

Substituting the solution in the equation of motion it can be proven that the oscillating damping coefficient hardly contributes to the motions. Because the low frequency oscillating coefficient has to be multiplied by the velocity, of which the dominant component consists of cos μt, the product results in a double frequency 2μ, which is beyond the resonance frequency of the low frequency surge motion, and so the contribution will be negligible. It must be mentioned that in principle a constant part will remain, which will affect the mean wave drift force slightly.

Neglecting the term with the oscillating coefficient the total wave drift force in a regular wave group will be:

$$X_{1t}^{(2)}(t) = \sum_{i=1}^{2}\sum_{j=1}^{2} \zeta_i\zeta_j T_{ij}\cos\left((\omega_i-\omega_j)t + (\varepsilon_i-\varepsilon_j) + \varepsilon_{ij}\right) + \sum_{i=1}^{2} \zeta_i^2 \frac{\partial T_{ii}}{\partial \dot{x}_1}\dot{x}_1 \quad \ldots \ldots (38)$$

Following the mentioned procedure the total wave drift force in irregular waves without current will be:

$$X_{1t}^{(2)} = \sum_{i=1}^{N}\sum_{j=1}^{N} \zeta_i\zeta_j T_{ij}\cos\left((\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)+\varepsilon_{ij}\right) + \sum_{i=1}^{N} \zeta_i^2 \frac{\partial T_{ii}}{\partial \dot{x}_1}\dot{x}_1 \quad \ldots \ldots (39)$$

<u>Total wave drift force in irregular waves combined with current</u>

Assuming that the vessel is sailing with the low speed U and performing a low frequency oscillation x_1 in a regular wave group the total wave drift force according to the gradient-method will yield:

$$X_t^{(2)}(\omega_1,\omega_2,U+\dot{x}_1,t) = \zeta_1^2 T_{11}^* + \zeta_2^2 T_{22}^* + 2\zeta_1\zeta_2 T_{12}^*\cos(\Delta\omega_{12}+\Delta\varepsilon_{12}+\varepsilon_{12}) + \zeta_1^2 D_{11}\dot{x}_1 + \zeta_2^2 D_{22}\dot{x}_1 \quad \ldots \ldots (40)$$

in which: $T_{11}^* = T_{11} + D_{11}U$

$T_{22}^* = T_{22} + D_{22}U$

$T_{12}^* = T_{12} + D_{12}U$

The formulation shows an increase of the constant parts and the oscillating part of the wave drift force caused by the low forward speed U.

The formulations mentioned so far have been based on a vessel sailing with low forward speed U in combination with low frequency oscillations, while the wave frequencies ω_1 and ω_2 are considered with regard to earth. If the vessel is moored in a current field with current speed V_c (bow directed into the current) the earth related wave frequencies should be transformed into the wave frequencies ω_e (= wave frequencies as measured in a fixed point in the Wave and Current Laboratory):

$$\omega_{1e} = \omega_1 + \frac{2\pi}{\lambda_1}V_c$$

$$\omega_{2e} = \omega_2 + \frac{2\pi}{\lambda_2}V_c \quad \ldots \ldots (41)$$

The total wave drift force in irregular waves combined with current with a velocity V_c is:

$$X_t^{(2)}(t) = \sum_{i=1}^{N}\sum_{j=1}^{N} \zeta_i\zeta_j T^*(\omega_i,\omega_j)\cos\left((\omega_i-\omega_j)t + (\varepsilon_i-\varepsilon_j) + \varepsilon_{ij}\right) + \sum_{i=1}^{N} \zeta_i^2 D_{ii}\dot{x}_1 \quad \ldots (42)$$

in which:

$$T^*(\omega_i,\omega_j) = T\left(\frac{\omega_i+\omega_j}{2}, \frac{\omega_i+\omega_j}{2}\right) + \frac{\partial T\left(\frac{\omega_i+\omega_j}{2}, \frac{\omega_i+\omega_j}{2}\right)}{\partial \dot{x}_1} V_c$$

while ω_i and ω_j stand for the frequency transformations according to:

$$\omega_i = \omega_{i0} + \frac{2\pi}{\lambda_{i0}} V_c$$

$$\omega_j = \omega_{j0} + \frac{2\pi}{\lambda_{j0}} V_c \quad \ldots\ldots\ldots (43)$$

in which ω_{i0} and ω_{j0} are the frequencies in still water and λ_{i0} and λ_{j0} are the corresponding wave lengths.

TIME-DOMAIN COMPUTATIONS IN IRREGULAR HEAD WAVES WITH AND WITHOUT CURRENT

Theory

For the time-domain computations of the low frequency motions of a moored tanker in irregular head waves with and without current the following equation of motion has to be solved in the time-domain:

$$\left(M+a_{11}(\mu_1)\right)\ddot{x}_1+B_{11}(\mu_1)\dot{x}_1+\bar{B}_1\dot{x}_1+c_{11}x_1 = X_1(t) \quad \ldots (44)$$

in which:

$a_{11}(\mu_1)$ = low frequency added mass coefficient
$B_{11}(\mu_1)$ = still water or current damping coefficient
$\bar{B}_1$ = mean wave drift damping coefficient
c_{11} = linear spring coefficient in surge direction
$X_1(t)$ = wave drift force in the time-domain
x_1 = $x_1^{(2)}$

The damping consists of the viscous and the mean wave drift damping. The viscous damping is either the still water or the current damping.

For the time-domain computation the wave drift force registration and the mean wave drift damping coefficient are necessary. To compute the registration and the damping coefficient respectively the wave train and the spectrum have to be known. It is assumed that for zero speed the transfer function of the wave drift forces $P(\omega_i,\omega_i)$ and the wave drift damping coefficient $D(\omega_i)$ are known. The transfer functions can be derived from either computations or model tests.

Applied to the wave spectrum the mean wave drift damping coefficient can be calculated:

$$\bar{B}_1(V_c) = 2\int_0^\infty S_\zeta(\omega)\; D(\omega^*) \quad \ldots\ldots\ldots (45)$$

in which: $S(\omega)$ = wave spectrum as measured in situ
ω^* = $\omega + kV_c$
k = wave number as function of ω
V_c = current velocity.
$D(\omega)$ = $Bl(\omega)/\zeta_a^2$

The transfer function of the wave drift force in current can be approximated by the gradient-method:

$$P(\omega_i^*, \omega_i^*) = P(\omega_i,\omega_i) - D(\omega_i)V_c \quad \ldots\ldots (46)$$

The time history of the wave drift force can be obtained by means of the quadratic impulse response function technique applied to the record of the measured wave train as proposed in ref. [14]. By means of the Fourier transform the quadratic impulse response can be written as follows:

$$g(\tau_1,\tau_2) =$$

$$= \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} G^{(2)}(\omega_1,\omega_2)\exp(i\omega_1\tau_1 - i\omega_2\tau_2)\; d\omega_1\; d\omega_2 \quad \ldots\ldots (47)$$

in which:

$G^{(2)}(\omega_1,\omega_2)$ = complex quadratic transfer function
= $P(\omega_1,\omega_2) + iQ(\omega_1,\omega_2)$
τ_1, τ_2 = time shifts

Because of the very low natural frequency of the system the matrix $G^{(2)}(\omega_1,\omega_2)$ may be composed by means of the in-phase components $P(\omega_1,\omega_2)$ approximated on base of $P(\omega_1,\omega_1)$ and $P(\omega_2,\omega_2)$, see eq. (25), while the quadrature component $Q(\omega_1,\omega_2)$ will be neglected.

On the base of linear interpolation of ω_i and ω_j in the matrices the Fourier transforms have been applied to obtain the quadratic impulse response functions $g^{(2)}(\tau_i,\tau_j)$.

Using the record of the adjusted waves $\zeta(t)$ the time-domain simulation of the wave drift force can be written as:

$$X_1(t) = \int_0^\infty\int_0^\infty g(\tau_1,\tau_2)\;\zeta(t-\tau_1)\;\zeta(t-\tau_2)\; d\tau_1\; d\tau_2 \quad . (48)$$

Using the theory, the wave drift force registration can be determined. After the determination of the viscous coefficients by means of model tests or the data given in ref. [14] the equation of motion can be solved.

Computed wave drift forces and mean wave drift damping coefficient

The quadratic transfer function for the wave drift force $P(\omega_i,\omega_i)$ for zero speed was computed for the loaded 200 kTDW tanker. The water depth corresponded to 82.5 m. The quadratic transfer function of the wave drift damping coefficient $D(\omega_i)$ was derived from the experiments as shown in Fig. 4. Based on the gradient-method the transfer functions of the wave drift force $P(\omega_i^*, \omega_i^*)$ and the wave drift damping coefficient $D(\omega_i^*)$ for 1.03 m/s current velocity were determined. The results are shown in Fig. 6.

With the knowledge of the transfer functions $P(\omega_i,\omega_i)$ and $P(\omega_i^*, \omega_i^*)$ the matrices of the quadratic transfer functions $P(\omega_i,\omega_j)$ and $P(\omega_i^*,\omega_j^*)$ respectively without and with current have been composed and are presented in Table 2. The impulse response function $g^{(2)}(\tau_i,\tau_j)$ has been determined for each of the matrices. For the deterministic approach the convolution has been applied to the calibrated wave trains as adjusted in the basin. For the spectra of the wave trains see section "model tests". The results of the computed wave drift force registration with and without current have been presented in the form of spectral densities over the frequency range of interest. The results are displayed in Fig. 7. Using the transfer functions of the wave drift damping as given in Fig.6 and applied to the spectra with and without current the following results for the mean wave drift damping coefficient were computed:

$B_1(V_c=0) = 48.2$ tf.s/m

$B_1(V_c=1.03$ m/s$) = 42.8$ tf.s/m

After obtaining the input data for the viscous damping coefficient the equation of motion can be solved.

Computed motions

The viscous damping coefficients have been obtained from the model tests. For the still water damping and the current damping the following values were obtained:

B_{11}(still water) $= 25.3$ tf.s/m

$B_{11}(V_c = 1.03$ m/s$) = 27.0$ tf.s/m

The total damping coefficient for the condition of waves without current is equal to 73.5 tf.s/m, while for the condition waves with current the total damping amounts to 69.8 tf.s/m. For the spring constant $c_{11} = 19.3$ tf/m was chosen. With these completed data the equations of motion were solved. The computed results are presented as time-domain plots in Fig. 8. While the simulation duration was 2.5 hours for full scale, only the last 1.5 hour has been presented. The first hour was necessary to overcome the transient phenomena of the computations.

MODEL TESTS

In order to correlate the computed registration of the wave drift force and the low frequency surge motion a series of model tests were carried out with the 200 kTDW tanker. The applied scale was 82.5. All model data were scaled to full scale values according to Froude's law of similitude.

The tests were carried out in the Wave and Current Laboratory of MARIN. The Wave and Current Laboratory measures 40 x 60 m. The basin consists of two sections with different water depths. One section has a water depth of 1 m. (40 x 35 m), while in the other one the depth amounts to 2 m (40 x 25 m). The tests have been carried out in the 1 m deep section. Over the width of 60 m of the basin current can be generated. Along the 40 m and along the 60 m sides of the basin wave generators have been installed.

For the present tests the wave generators along the side were used. Prior to the wave calibration a homogeneously distributed current field was adjusted with a velocity corresponding to 2 kn for full scale. Combined with current or without current the same wave spectra were adjusted at the projected location of the C.o.G. of the moored tanker. Each sea state was prepared for a test duration of 2.5 hours for full scale time. The wave spectra are shown in Fig. 9.

Both wave drift and motion measurements were carried out. For the wave drift force measurements a vertically positioned cylinder supported by air lubricated bearings was used to keep the tanker on station. The bearings were earth fixed. As is illustrated in Fig. 10 the test set-up gives freedom to heave and pitch motions. As a consequence of the set-up the tanker was not able to perform high-freqency surge motions. This approach is allowed, because it can be shown that the contribution of the high frequency surge motion to the wave drift forces is negligibly small. A force transducer was mounted to the lower side of the cylinder measuring the force in the horizontal direction. The measured horizontal force consisted of the first order and the second order wave forces. By means of a low pass filtering technique the low frequency second order wave drift forces were obtained. In terms of spectral density the measured drift forces are presented in Fig. 7.

For the surge motion measurements the tanker was kept on station by a spring system, see Fig. 11, having a linear spring constant in surge drection $c_{11} = 19.3$ tf/m. Prior to the tests in waves, extinction tests were carried out in still water and in current. The results of the extinction tests in terms of the logarithmic decrement are shown in Fig. 12. The force in 1.03 m/s current amounts to 13.1 tf. During the tests the surge motion was measured in the centre of gravity (C.o.G.) by means an optical tracking device. The result in the form of time traces is shown in Fig. 8.

CONCLUSIONS

By means of 3-D diffraction potential theory the wave drift forces for small values of forward speed can be computed. By means of the derivative of the wave drift force for zero speed to forward speed the wave drift damping coefficient can be determined. A reasonable good agreement was found with the experiments.

The gradient-method was used to determine the main diagonal of the matrix of the quadratic transfer function of the wave drift force in current. An approximation was applied to compose the off-diagonals. In terms of spectral densities a reasonable good agreement was found between the computed and the experimentally determined wave drift force for the frequency range of interest.

By means of the transfer function of the wave drift damping coefficients and the wave spectra the mean wave damping coefficients were derived. With regard to the viscous damping coefficient the total damping was increased by appoximately a factor 3 for both the still water and the current condition.

By applying the quadratic impulse response technique a reasonable good agreement was found between the computed and the measured low frequency motions.

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TABLE 1—THE PARTICULARS OF THE TANKER

Designation	Symbol	Unit	Magnitude loaded
Draft in percent of loaded draft			100%
Length between perpendiculars	L	m	310.00
Breadth	B	m	47.17
Depth	H	m	29.70
Draft	T	m	18.90
Wetted area	S	m^2	22,804
Displacement volume	∇	m^3	234,994
Mass vessel	m	tfs^2/m	24,553
Centre of bouyancy forward of section 10	$\overline{FB}$	m	6.6
Centre of gravity above keel	$\overline{KG}$	m	13.32
Metacentric height transverse	$\overline{GM}_t$	m	5.78
Metacentric height longitudinal	$\overline{GM}_l$	m	403.83
Transverse radius of gyration in air	k_{11}	m	14.77
Transverse radius of gyration in water			17.02
Longitudinal radius of gyration in air	k_{22}	m	77.47
Yaw radius of gyration in air	k_{66}	m	79.30

TABLE 2—MATRICES OF THE QUADRATIC TRANSFER FUNCTIONS OF THE WAVE DRIFT FORCES WITH AND WITHOUT CURRENT

T_{ij} (u=0 m/s)

$\omega_j \backslash \omega_i$	0.12	0.16	0.20	0.24	0.28	0.32	0.36	0.40	0.44	0.48	0.52	0.56	0.60	0.64	0.68	0.72	0.76	0.80	0.84	0.88	0.92	0.96	1.00	1.04
0.12	0.76	0.59	0.43																					
0.16		0.43	0.34	0.25																				
0.20			0.25	0.36	0.47																			
0.24				0.47	0.65	0.82																		
0.28					0.82	1.11	1.40																	
0.32						1.40	1.81	2.22																
0.36							2.22	3.35	4.47															
0.40								4.47	5.59	6.71														
0.44									6.71	8.06	9.41													
0.48										9.41	10.79	12.16												
0.52											12.16	12.62	13.09											
0.56												13.09	13.49	13.88										
0.60													13.88	12.92	11.95									
0.64														11.95	10.96	10.02								
0.68															10.02	9.31	8.60							
0.72																8.60	8.71	8.92						
0.76																	8.92	9.75	8.23					
0.80																		9.23	9.11	9.00				
0.84																			9.00	8.86	8.72			
0.88																				8.72	8.72	8.72		
0.92																					8.72	8.76	8.79	
0.96																						8.79	8.83	8.87
1.00																							8.87	8.90
1.04																								8.95

$$T(\omega_i,\omega_j) = T(\frac{\omega_i+\omega_j}{2},\frac{\omega_i+\omega_j}{2})$$

T_{ij} (u=1.03 m/s)

$\omega_j \backslash \omega_i$	0.12	0.16	0.20	0.24	0.28	0.32	0.36	0.40	0.44	0.48	0.52	0.56	0.60	0.64	0.68	0.72	0.76	0.80	0.84	0.88	0.92	0.96	1.00	1.04
0.12	0.82	0.66	0.50																					
0.16		0.50	0.39	0.28																				
0.20			0.28	0.42	0.56																			
0.24				0.56	0.72	0.89																		
0.28					0.89	1.29	1.70																	
0.32						1.70	2.11	2.51																
0.36							2.51	3.74	4.98															
0.40								4.98	6.47	7.96														
0.44									7.96	9.49	11.02													
0.48										11.02	12.61	14.21												
0.52											14.21	15.50	16.78											
0.56												16.78	16.79	16.80										
0.60													16.80	16.76	16.71									
0.64														16.71	15.58	14.44								
0.68															14.44	13.31	12.18							
0.72																12.18	11.22	10.27						
0.76																	10.27	10.26	10.24					
0.80																		10.24	10.40	10.56				
0.84																			10.56	10.68	10.79			
0.88																				10.79	10.77	10.75		
0.92																					10.75	10.73	10.71	
0.96																						10.71	10.77	10.82
1.00																							10.82	10.90
1.04																								10.98

$$T(\omega_i,\omega_j) = T(\frac{\omega_i + \omega_j}{2},\frac{\omega_i + \omega_j}{2})$$

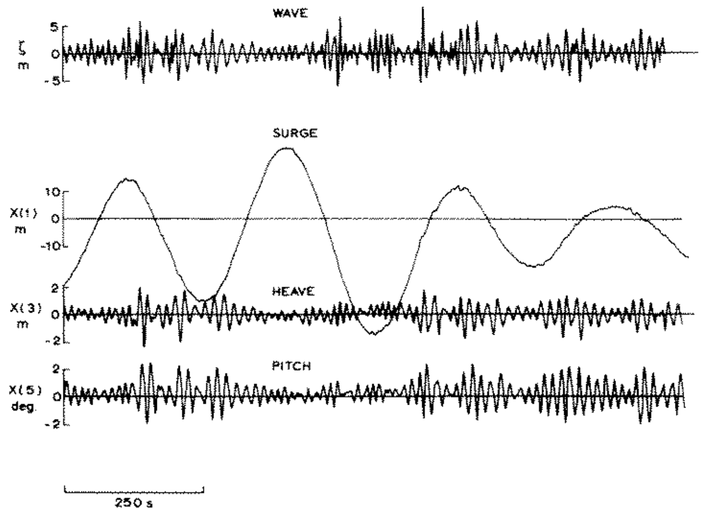

Fig. 1—A registration of the motions of a moored VLCC in head waves.

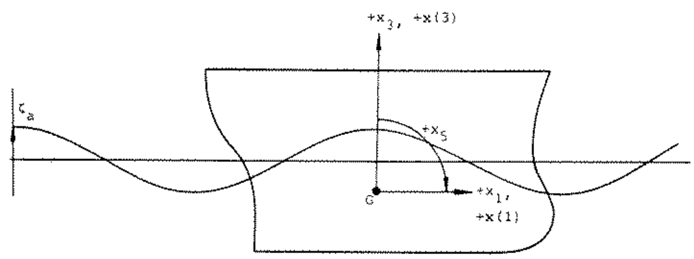

Fig. 2—System of co-ordinates.

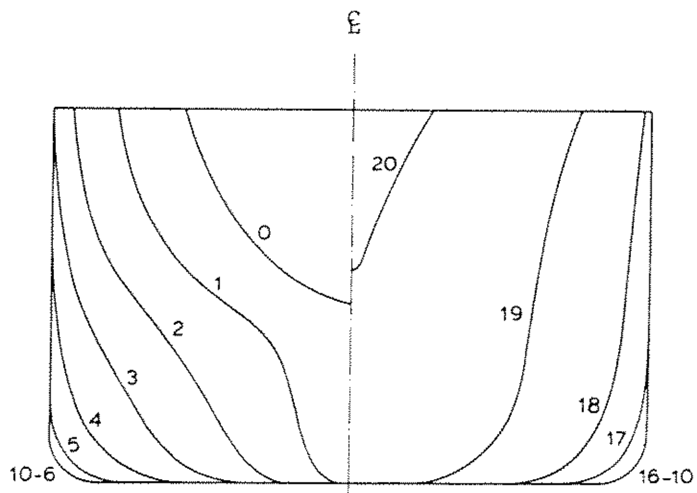

Fig. 3—Body plan.

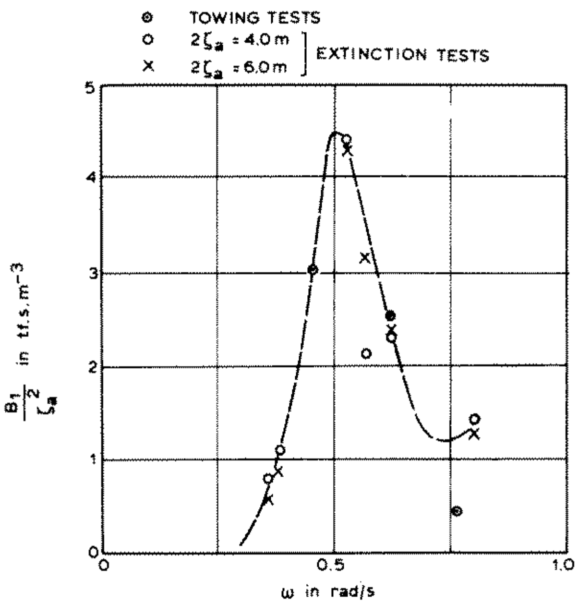

Fig. 4—Experimentally derived values of the wave drift damping quadratic transfer function.

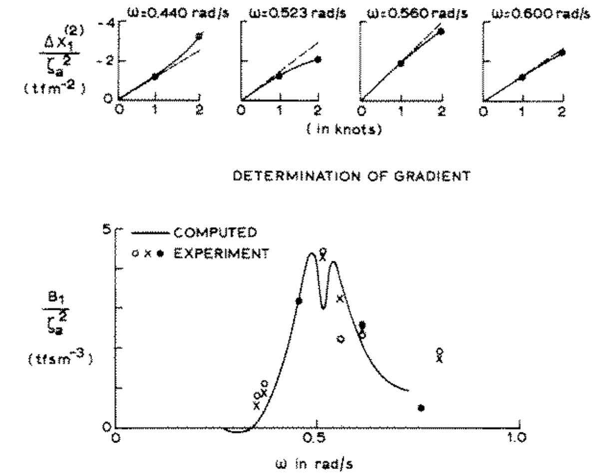

Fig. 5—Quadratic transfer function of the wave drift damping coefficient for the 200-kTDW tanker in head waves (earth-bound wave frequencies).

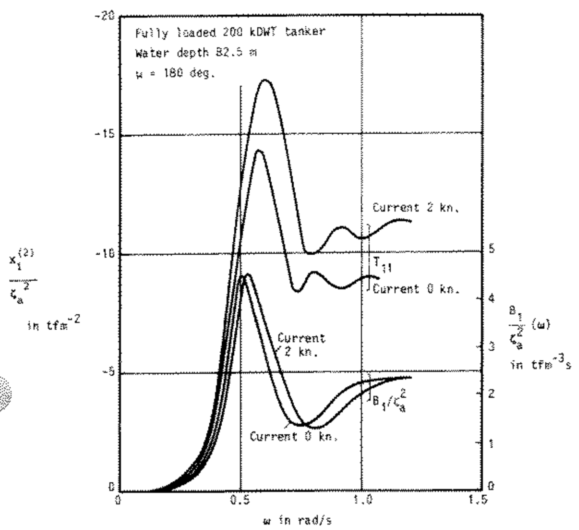

Fig. 6—The quadratic transfer functions of the wave drift force and wave drift damping coefficients with and without 2-kn current.

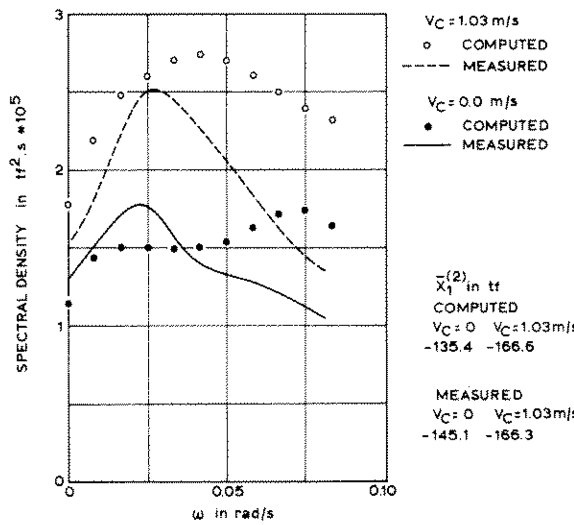

Fig. 7—The spectral densities of the measured and computed wave drift forces.

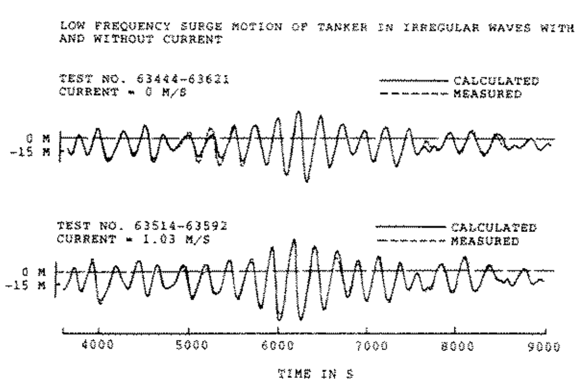

Fig. 8—The computed and measured low-frequency surge motion of the tanker in irregular waves with and without current.

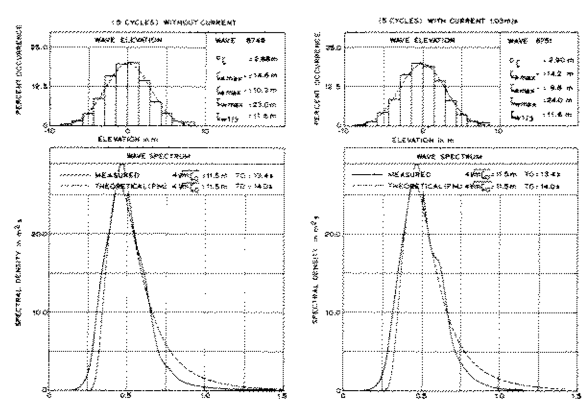

Fig. 9—The adjusted wave spectra with and without current.

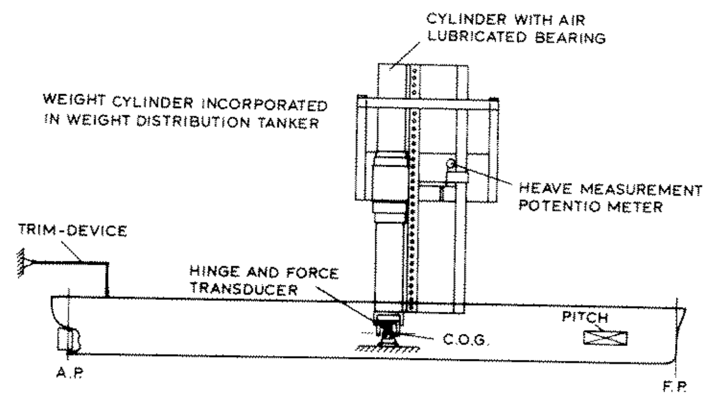

Fig. 10—Test setup for the wave drift force measurement.

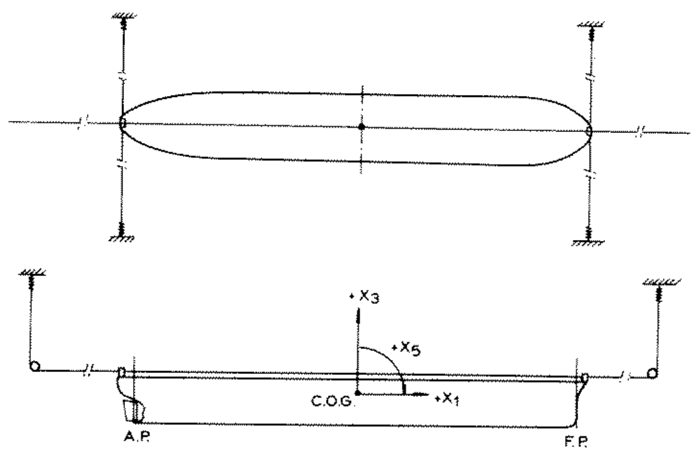

Fig. 11—Test setup.

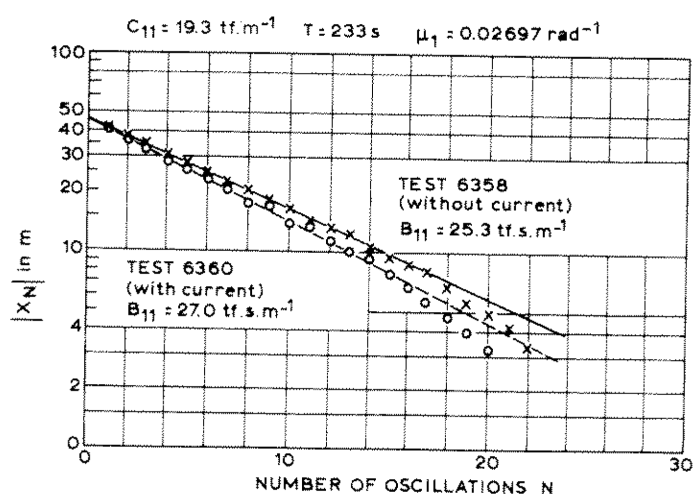

Fig. 12—Surge extinction test with and without current.