

17

PRADS '87

22-26 June, 1987

The Proceedings of the
Third International Symposium on
Practical Design of Ships and Mobile Units

Trondheim, Norway





Maritime Research Institute Netherlands

2, Huismanweg, PO Box 26
6700 AA Wageningen, The Netherlands
Telephone +31 (0)31 8371 9371 Telex 45145 nrdnl n
Telex +31 8370 93100

CONSIDERATIONS ON WAVE DRIFT
DAMPING OF A MOORED TANKER
FOR ZERO AND NON-ZERO DRIFT
ANGLES

By: R.H.M. Huijsmans
J.E.W. Wichers

CONSIDERATIONS ON WAVE DRIFT DAMPING
OF A MOORED TANKER FOR ZERO AND
NON-ZERO DRIFT ANGLES

R.H.M. Huijsmans
J.E.W. Wichers
Project Managers

Maritime Research
Institute Netherlands
(MARIN)

The Netherlands

Abstract

This paper is concerned with the development and application of hydrodynamic theory required for the prediction of wave drift damping phenomena as introduced by Wichers in 1979 and 1982, ref. (1) and (2). The concept of wave drift damping necessitates the development of forward speed dependent expressions for the mean wave drift forces and a new 3-d diffraction potential algorithm for forward speed (Wichers and Huijsmans, ref. (3) and Hearn and Koon, Ref. (4)).

In this paper the first order calculated motion response functions for heave and pitch for a 200 kDWT tanker moored in deep water waves with current are compared with results of model test experiments.

Also for this situation the computed mean wave drift forces are validated with results from model tests as reported by Wichers, ref. (5).

Results will also be discussed of the case in which the current is not head on, but at a certain drift angle; in this situation current and waves are coming from the same direction.

1. Introduction

In 1971 Remery and Hermans ref. (6) reported results of the excitation and motion of a barge moored in wave groups. They considered the surge motion only and showed that excitation of the large amplitude low frequency motion was due to the low frequency drift

$$\begin{aligned}
& \tilde{\phi}_{tt} + g\tilde{\phi}_z + 2U\tilde{\phi}_{xt} + 2\nabla\bar{\phi}\cdot\nabla\tilde{\phi}_t + \\
& + (U^2 + 2U\bar{\phi}_x + \bar{\phi}_x^2)\tilde{\phi}_{xx} + 2(U + \bar{\phi}_x)\bar{\phi}_y\tilde{\phi}_{xy} + \\
& + \bar{\phi}_y^2\tilde{\phi}_{yy} + (3U\bar{\phi}_{xx} + \bar{\phi}_x\bar{\phi}_{xx} + \bar{\phi}_y\bar{\phi}_{xy})\tilde{\phi}_x + \\
& + (2U\bar{\phi}_{xy} + \bar{\phi}_x\bar{\phi}_{xy} + \bar{\phi}_y\bar{\phi}_{yy})\tilde{\phi}_y + L^{(2)}\{\tilde{\phi}\} = 0 \quad \text{at } z=0
\end{aligned} \tag{5}$$

The boundary condition on the hull can be written in a similar way for all radiating and diffracted modes. Generally we have the condition:

$$(n\cdot\nabla\tilde{\phi}) = [\dot{\alpha} + \nabla x (\alpha x \bar{W})] \cdot n \tag{6a}$$

at the mean position of the hull, in which α is the oscillation motion and $\bar{W} = (u, v, 0)^T$. The speed components will be:

$$u = |\bar{W}| \cos\beta$$

$v = |\bar{W}| \sin\beta$, with β the drift angle of the ship with respect to the current as defined in Figure 1.

For the six modes of motion the body boundary condition now reads:

$$\frac{\partial}{\partial n} \phi_k = i \omega_e n_k + \bar{W} m_k \quad k = 1, (1), 6 \tag{6b}$$

in which:

$$\bar{W} m_k = -(n\cdot\nabla) w = 0 \quad k = 1, 2, 3$$

$$\bar{W} m_k = -(n\cdot\nabla) (\underline{r} \times w) \quad k = 4, 5, 6$$

which leads to:

$m_4 = -\sin\beta n_2$; $m_5 = \cos\beta n_3$; $m_6 = \sin\beta n_1 - \cos\beta n_2$ with n_k is the direction cosine.

The non-linear operator on $\tilde{\phi}$ will be neglected as well. The first line in (5) contains linear terms in U . Our Ansatz is that in order to obtain the first order approximation with respect to U the second order terms with respect to U may be neglected in the free surface. In the next section we show that in general this is true, but first we discuss the construction of the regular part of the perturbation problem, with the complete linear free surface condition.

We assume $\tilde{\phi}(\underline{x}, t; U)$ to be oscillatory:

$$\tilde{\phi}(\underline{x}, t; U) = \phi(\underline{x}, U) e^{-i\omega t} \quad (7)$$

The free surface condition is written as:

$$-\omega^2 \phi - 2i\omega U \phi_x + U^2 \phi_{xx} + g \phi_z = D(U; \bar{\phi})\{\phi\} \quad \text{at } z = 0 \quad (8)$$

where $D(U; \bar{\phi})$ is a linear differential operator acting on ϕ as defined in (5). We apply Green's theorem to a problem in D_i inside S and to the problem in D_e outside S where S is the ship's hull. The potential function inside S obeys condition (8) with $D = 0$, while the Green's function fulfills the homogeneous adjoint free surface condition:

$$-\omega^2 G + 2i\omega U G_\xi + U^2 G_{\xi\xi} + g G_\xi = 0 \quad \text{at } z=0 \quad (9)$$

This Green's function has the form:

$$G(\underline{x}, \underline{\xi}; U) = -\frac{1}{r} + \frac{1}{r_1} - \psi(\underline{x}, \underline{\xi}; U) \quad (10)$$

where $r = |\underline{x} - \underline{\xi}|$ and $r_1 = |\underline{x} - \underline{\xi}'|$, where $\underline{\xi}'$ is the image of $\underline{\xi}$ with respect to the free surface.

Combining the formulation inside and outside the ship we obtain a description of the potential function defined outside S by means of a source and vortex distribution of the following form:

$$\begin{aligned} & - \iint_S \gamma(\underline{\xi}) \frac{\partial}{\partial n} G(\underline{x}, \underline{\xi}) dS_\xi - \iint_S \sigma(\underline{\xi}) G(\underline{x}, \underline{\xi}) dS_\xi - \frac{2i\omega U}{g} \int_{WL} \gamma(\underline{\xi}) G(\underline{x}, \underline{\xi}) dn \\ & + \frac{U^2}{g} \int_{WL} [\gamma(\underline{\xi}) \frac{\partial}{\partial \xi} G(\underline{x}, \underline{\xi}) - \{\alpha_t \gamma_t(\underline{\xi}) + \alpha_T \gamma_T(\underline{\xi})\} G(\underline{x}, \underline{\xi})] dn \\ & + \frac{U^2}{g} \int_{WL} \alpha_n \sigma(\underline{\xi}) G(\underline{x}, \underline{\xi}) dn + \frac{i\omega}{g} \iint_{FS} G(\underline{x}, \underline{\xi}) D\{\phi\} dS_\xi = 4\pi \phi(\underline{x}) \end{aligned} \quad (11)$$

$\alpha_t = \cos(\theta_{x,t})$, $\alpha_T = \cos(\theta_{x,T})$, $\alpha_n = \cos(\theta_{x,n})$ where n is the normal and t the tangent to the waterline and $T = t \times n$ the bi-normal.

It is clear that with the choice $\gamma(\underline{\xi}) = 0$ the integral along the waterline gives no contribution up to order U . The source distribution we obtain this way is not a proper source distribution, because it expresses the function ϕ in a source distribution along the free surface with strength proportional to the derivatives of the same function ϕ . However, this formulation is linear in U and moreover the integrand tends to zero rapidly for increasing

force. At the time they used in their simplified model a non realistic large damping coefficient to predict the low frequency motion response. In 1980 Pinkster, ref. (7), published techniques resulted in excellent agreement of the calculated drift forces at zero forward speed with experiments. An unsolved problem, however, is the estimation of the motion of a moored ship, especially when the low frequency motions have large amplitudes.

Wichers and Huijsmans, ref. (8), showed that the damping at the natural frequencies of the mooring system have to be considered carefully. Results of model test experiments showed that a large part of the damping at these natural frequencies could be attributed to the velocity dependency of the wave drift forces. Hence, the effect of moderate speed should be accounted for if one wants to evaluate this wave damping phenomenon.

However, the introduction of forward speed into ship motions is not a trivial task. A direct approach is reported by Inglis, ref. (9), M. Chang, ref. (11) and J. Bougis, ref. (10). However, this approach is very time consuming. Another defect is the improper treatment of the free surface. We shall develop a consistent method based on a perturbation with respect to small values for the Froude number.

2. Mathematical formulation

The total potential function will be split in a steady and a non-steady part in a well-known way:

$$\phi(\underline{x}, t) = Ux + \bar{\phi}(\underline{x}; U) + \tilde{\phi}(\underline{x}, t; U) \quad (1)$$

in this formulation U is the incoming unperturbed velocity field, obtained by considering a coordinate system fixed to a ship moving under a drift angle β . In our approach this angle need not be small. The time dependent part of the potential consists of an incoming wave at frequency ω , a diffracted and/or a radiated wave contribution. To compute the drift force all these components will be taken into account. In this paper we restrict ourselves to a general theory concerning the wave components. Some results of drift force computations are shown as well.

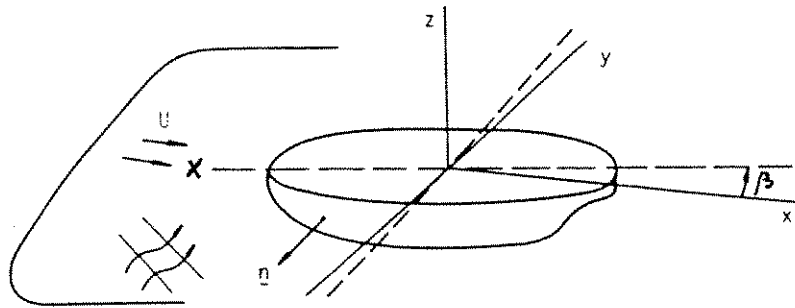


Fig. 1 System of axis

The equations for the total potential ϕ can be written as:

$$\Delta\phi = 0 \text{ in the fluid domain } D_e \quad (2)$$

At the free surface we have the dynamic and kinematic boundary condition.

$$\left. \begin{aligned} g\zeta + \phi_t + \frac{1}{2} \nabla\phi \cdot \nabla\phi &= \text{const.} \\ \phi_z - \phi_x \zeta_y - \phi_y \zeta_x - \zeta_t &= 0 \end{aligned} \right\} \text{ at } z = \zeta(x, y, t) \quad (3)$$

We assume that the waves are high compared with the Kelvin wave pattern and that they both are small, hence the free surface condition can be expanded at $z = 0$. Elimination of ζ leads to the following non-linear condition:

$$\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial z} + \frac{\partial}{\partial t} (\nabla\phi \cdot \nabla\phi) + \nabla\phi \cdot \nabla \left(\frac{\nabla\phi \cdot \nabla\phi}{2} \right) = 0 \text{ at } z = 0 \quad (4)$$

To compute the wave resistance at low speed the free surface must be treated more carefully, because the wave height is of asymptotically smaller order. This problem is studied extensively by Eggers, Ref (12), Baba, ref. (13), Hermans, ref. (14) and Brandsma, ref. (15). The velocity field is well described by the double body potential with a small wave pattern. Therefore we take the double body potential into account and we neglect the stationary wave pattern.

For the wave potential $\tilde{\phi}(\underline{x}, t; U)$ the free surface condition now becomes:

distance R. So finally we arrive at the formulation:

$$\begin{aligned}
 & - 2\pi\sigma(\underline{x}) - \iint_S \sigma(\underline{\xi}) \frac{\partial}{\partial n_x} G(\underline{x}, \underline{\xi}) dS_\xi + \frac{U^2}{g} \int_{WL} \alpha_n \sigma(\underline{\xi}) \frac{\partial}{\partial n} G(\underline{x}, \underline{\xi}) d\eta \\
 & + \frac{i\omega}{g} \iint_{FS} \frac{\partial}{\partial n_x} G(\underline{x}, \underline{\xi}) D\{\phi\} dS_\xi = 4\pi V(\underline{x}), \quad x \in S
 \end{aligned} \tag{12}$$

and

$$\begin{aligned}
 4\pi\phi(\underline{x}) = & - \iint_S \sigma(\underline{\xi}) G(\underline{x}, \underline{\xi}) dS_\xi + \frac{U^2}{g} \int_{WL} \alpha_n \sigma(\underline{\xi}) G(\underline{x}, \underline{\xi}) d\eta \\
 & + \frac{i\omega}{g} \iint_{FS} G(\underline{x}, \underline{\xi}) D\{\phi\} dS_\xi, \quad x \in D_e
 \end{aligned} \tag{13}$$

We now consider small values of U, keeping in mind that there are two dimensionless parameters that play a role in the limit. We consider $\tau = \frac{U\omega}{g} \ll 1$ and $\nu = \frac{gL}{U^2} \gg 1$. It turns out that the source strength and the potential function can be expanded as follows:

$$\begin{aligned}
 \sigma(\underline{x}; U) &= \sigma_0(x) + \tau\sigma_1(\underline{x}) + \tilde{\sigma}(\underline{x}; U) \\
 \phi(\underline{x}; U) &= \phi_0(x) + \tau\phi_1(x) + \tilde{\phi}(x; U)
 \end{aligned} \tag{14}$$

where $\tilde{\sigma}$ and $\tilde{\phi}$ are $O(\tau^2)$ as $\tau \rightarrow 0$, while the expansion of the Green's function is less trivial.

3. The Green's function

In this section we present an asymptotic expansion of the Green's function. The Green's function follows from the source function presented in Wehausen and Laitone, see ref. (16).

In the case $\tau < 1/4$ the function $\psi(\underline{x}, \underline{\xi}; U)$ is written as follows:

$$\psi(\underline{x}, \underline{\xi}; U) = \frac{2g}{\pi} \int_0^{\pi/2} d\theta \int_{L_1} dk F(\theta, k) + \frac{2g}{\pi} \int_{\pi/2}^{\pi} d\theta \int_{L_2} dk F(\theta, k) \tag{15}$$

where:

$$F(\theta, k) = \frac{k \exp(k[z + \zeta + i(x-\xi)\cos\theta]) \cos[k(y-\eta)\sin\theta]}{gk - (\omega + kU\cos\theta)^2} \tag{16}$$

The contours L_1 and L_2 are given as follows:

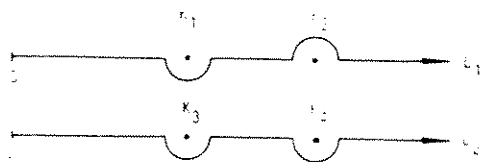


Fig. 2

These contours are chosen such that the 'radiation' conditions are satisfied. The radiated waves are outgoing and the Kelvin pattern is behind the ship. The values k are the poles of $F(\theta, k)$. For small values of τ these poles behave as follows:

$$\sqrt{gk_1}, \sqrt{gk_3} \sim \omega + 0(\tau) \quad \text{as } \tau \rightarrow 0 \quad (17)$$

$$\sqrt{gk_2}, -\sqrt{gk_4} \sim \frac{\omega}{\tau \cos \theta} + 0(1) \quad \text{as } \tau \rightarrow 0 \quad (18)$$

A careful analysis of the asymptotic behaviour of $\psi(\underline{x}, \underline{\xi}; U)$ for small values of U leads to a regular part and an irregular part:

$$\psi(\underline{x}, \underline{\xi}; U) = \psi_0(\underline{x}, \underline{\xi}) + \tau \psi_1(\underline{x}, \underline{\xi}) + \dots + \tilde{\psi}_0(\underline{x}, \underline{\xi}) + \nu^{-1} \tilde{\psi}_1(\underline{x}, \underline{\xi}) + \dots \quad (19)$$

where

$$\psi_0(\underline{x}, \underline{\xi}) = 2g \int_{L_2} \frac{k e^{k(z+\zeta)}}{gk - \omega^2} J_0(kR) dk \quad (20)$$

$$\psi_1(\underline{x}, \underline{\xi}) = 4ig^2 \cos \theta' \int_{L_2} \frac{k^2 e^{k(z+\zeta)}}{(gk - \omega^2)^2} J_1(kR) dk \quad (21)$$

where $R^2 = (x-\xi)^2 + (y-\eta)^2$ and $\theta' = \text{arctg} \left(\frac{y-\eta}{x-\xi} \right)$ and

$$\begin{aligned} \tilde{\psi}_0(\underline{x}, \underline{\xi}) = & -4\nu \int_0^{\pi/2} \exp[\nu(z+\zeta)\sec^2\theta] \sin[\nu(x-\xi)\sec\theta] * \\ & * \cos[\nu(y-\eta)\sin\theta\sec^2\theta] \sec^2\theta d\theta \end{aligned} \quad (22)$$

In Hermans and Huijsmans, ref. (17), it is shown that due to the highly oscillatory nature, the influence of (22) may be neglected in our first order correction for small values of τ .

4. Expansion of the source strength

In this section an approximation solution of (12) will be derived. Inserting (14) and (20) in (12) one obtains for like powers of τ the following set of equations:

$$-2\pi \sigma_0(\underline{x}) - \iint_S \sigma_0(\underline{\xi}) \frac{\partial G_0}{\partial n_x}(\underline{x}, \underline{\xi}) dS_\xi = 4\pi V_0(\underline{x}), \quad x \in S$$

and

$$\begin{aligned} -2\pi \sigma_1(\underline{x}) - \iint_S \sigma_1(\underline{\xi}) \frac{\partial G_0}{\partial n_x}(\underline{x}, \underline{\xi}) dS_\xi = & -\iint_S \sigma_0(\underline{\xi}) \frac{\partial}{\partial n_x} \psi_1(\underline{x}, \underline{\xi}) dS_\xi + \\ + \frac{2i}{g} \iint_{FS} \frac{\partial G_0}{\partial n_x}(\underline{x}, \underline{\xi}) \nabla \bar{\phi} \cdot \nabla \phi_0 ds_\xi + & 4\pi V_1(\underline{x}) \end{aligned} \quad (24)$$

where $G_0(\underline{x}, \underline{\xi}) = -\frac{1}{r} + \frac{1}{r_1} - \psi_0(\underline{x}, \underline{\xi})$ is the zero speed pulsating wave source, and $V(\underline{x}) = V_0(\underline{x}) + \tau V_1(\underline{x}) + O(\tau^2)$.

This perturbation leads to a fast algorithm that takes into account speed effects once a fast method is available for the zero speed diffraction problem. At MARIN the diffraction program has been extended with the Fingreen subroutines of Newman.

The potential functions (14) now become:

$$\phi_0(\underline{x}) = -\frac{1}{4\pi} \iint_S \sigma_0(\underline{\xi}) G_0(\underline{x}, \underline{\xi}) dS_{\underline{\xi}}$$

and

$$\begin{aligned} \phi_1(\underline{x}) = & \frac{1}{4\pi} \iint_S \sigma_0(\underline{\xi}) \psi_1(\underline{x}, \underline{\xi}) dS_{\underline{\xi}} - \frac{1}{4\pi} \iint_S \sigma_1(\underline{\xi}) G_0(\underline{x}, \underline{\xi}) dS_{\underline{\xi}} + \\ & + \frac{i}{2\pi g} \iint_{FS} G_0(\underline{x}, \underline{\xi}) \nabla \bar{\phi} \cdot \nabla \phi_0 dS_{\underline{\xi}} \end{aligned} \quad (25)$$

5. Mean wave drift forces

In order to calculate the mean wave drift forces a pressure integration technique is used. The validity of this approach has been elucidated by Pinkster, ref. (7), for the zero speed case. The resulting expression is:

$$\begin{aligned} F^{(2)} = & -g \int_{WL} \frac{1}{2} \rho |\zeta_r^{(1)}|^2 \underline{n} d l + \alpha^{(1)} \times (M \cdot \ddot{\underline{X}}_g^{(1)}) + \\ & + \iint_{S_0} \frac{1}{2} \rho |\nabla \tilde{\phi}^{(1)}|^2 \underline{n} dS + \iint_{S_0} \rho(\underline{x}^{(1)} \cdot \nabla \tilde{\phi}_t^{(1)}) \underline{n} dS \end{aligned} \quad (26)$$

In this expression the velocity dependent potentials as derived in the previous section have to be substituted.

It will be clear that we cannot make a perturbation series with respect to τ . The first order motion $\underline{X}^{(1)}$ depends on U in a complicated way, phase and amplitude are influenced by the small parameter τ . Therefore it is not possible to express (26) in a power series in τ uniformly. Similar arguments hold for some of the other terms. So, finally, the effect of the speed on the drift forces is computed by evaluating (26) numerically.

6. Results of computations and model test experiments

At MARIN a number of tests have been conducted. The tests comprise in short regular wave tests on a stationary moored 200 kDWT tanker in 82.5 m water depth with waves and current parallel to each other. Particulars of the tanker are displayed in Table 1.

Table 1 Particulars of the tanker

Designation	Symbol	Unit	100% T
Length between perpendiculars	L_{pp}	m	310
Breadth	B	m	47.7
Draft	T	m	18.90
Displacement weight	Δ	T	240,869
Displacement volume	V	m ³	234,826
Centre of gravity fore St. 10	LCG	m	6.61
Centre of gravity above base	\overline{KG}	m	13.32
Metacentric height:			
Transverse	\overline{GM}_T	m	5.78
Longitudinal	\overline{GM}_L	m	404.12
Radius of gyration in air:			
Transverse	k_{xx}	m	14.77
Longitudinal	k_{yy}	m	77.47
Vertical	k_{zz}	m	79.30
Transverse gyration in water	k_{xx}	m	17.00
Roll period	T_ϕ	s	14.20

One case concerns the head on current and waves and the other involves the current and waves at an angle of 135° to the ship, see Fig. 3.

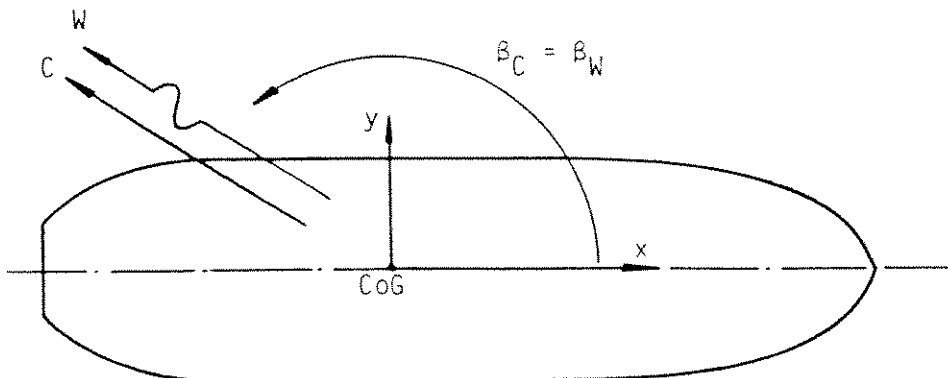


Fig. 3 Definition of wave and current direction with regard to the vessel

The results of model tests and computations for head waves and current have been extensively described by Wichers, ref. (5) and Huijsmans, ref. (18). The motion response in head waves with zero and 2 m/s current speed has been given in Fig. 4. The results of the mean wave drift forces are shown in Fig. 5. The derived wave drift damping coefficients are displayed in Fig. 6. The results of model tests for the tanker with the co-linear directed waves and current under a heading of 135° are presented in Figures 7 and 8. The results concern the mean surge and sway wave drift forces in zero and 2 m/s current speed.

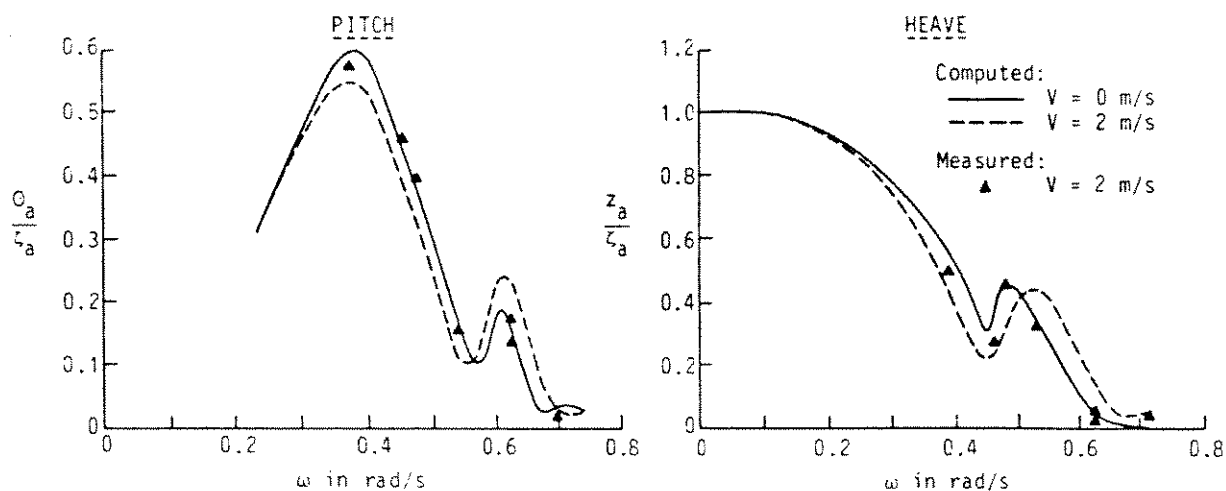


Fig. 4 Response of a 200 kWDT tanker in head waves

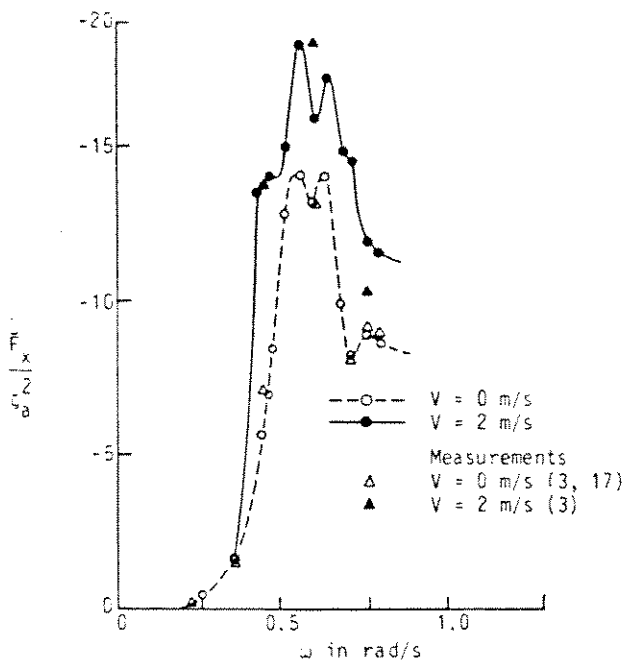


Fig. 5 Mean surge drift force in head waves for a 200 kWDT tanker

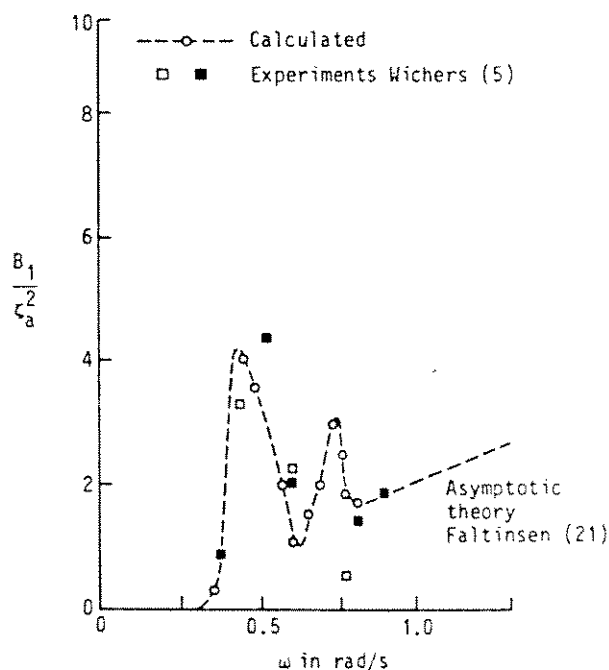


Fig. 6 "Wave drift damping" coefficient in head waves for a 200 kWDT tanker

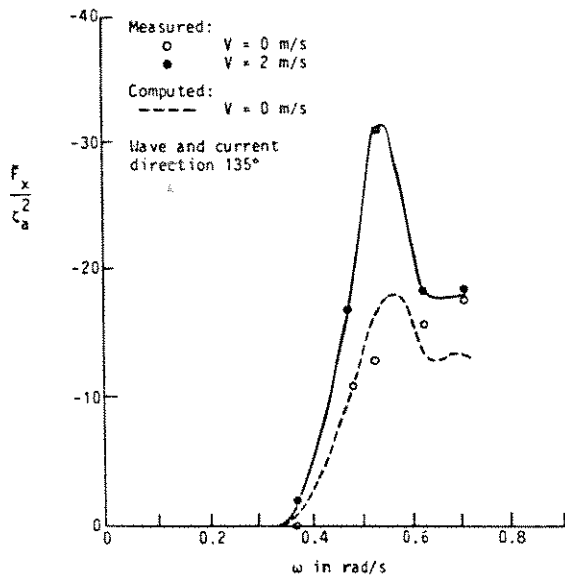


Fig. 7 Mean surge drift force in waves and current under a heading of 135° for a 200 kDWT tanker

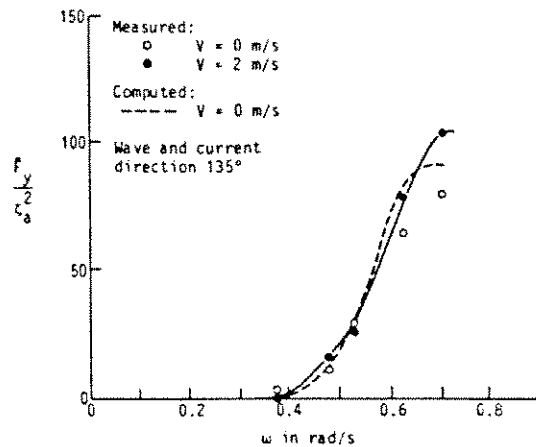


Fig. 8 Mean transverse drift force in waves and current under a heading of 135° for a 200 kDWT tanker

7. Discussion

For the case of zero drift angle the contribution of the free surface integrals will be negligible. This is due to the fact that in the case of zero drift angle the double body stationary potential may be neglected. However, in the case of non-zero drift angle this is no longer true and computations have to be adjusted to take into account the effect of the double body stationary potential. Results of these computations will be presented in the near future. Also the nature of the wave drift damping appeared from the model tests with non-zero drift angle. The surge drift force (Fig. 7) has a large speed dependency in the wave frequency range of 0.4 rad/s to 0.7 rad/s. This indicates that also in this situation "wave drift damping" is an important phenomena for the surge motion of the tanker. However, the sway drift force does not display such a velocity dependency, which allows the conclusion that in this case, the sway motion will not be largely influenced by the presence of wave drift damping, see Fig. 8. Here other effects play a more important role as has been reported by Faltinsen et al., ref. (19) and Sortland, ref. (20).

References

- (1) WICHERS, J.E.W., On the low frequency surge motions of vessels mooring in high seas, Offshore Technology Conference, Paper 4437, Houston, 1982.
- (2) WICHERS, J.E.W. and VAN SLUIJS, M.F., The influence of waves on the low frequency hydrodynamic coefficients of moored vessels, Offshore Technology Conference, Paper 2625, Houston, 1979.
- (3) WICHERS, J.E.W. and HUIJSMANS, R.H.M., On the low frequency hydrodynamic damping forces acting on offshore moored vessels, Offshore Technology conference, Paper 4813, Houston, 1984.
- (4) HEARN, G. AND KOON, T., Second order fluid damping, Progress Report, July 1985, University of Newcastle.
- (5) WICHERS, J.E.W., Progress in computer simulations of SPM moored vessels, Offshore Technology Conference, Paper 5175, Houston, 1986.
- (6) REMERY, G.F.M. and HERMANS, A.J., The slow drift oscillations of a moored object in random seas", Offshore Technology Conference, Paper 1500, Houston, 1971.
- (7) PINKSTER, J.A., Low frequency second order wave exciting forces on floating structures, Ph.D. Thesis, Report 650, MARIN (N.S.M.B.), 1980.
- (8) WICHERS, J.E.W. and HUIJSMANS, R.H.M., On the low frequency hydrodynamic damping forces acting on offshore moored vessels, Offshore Technology Conference, Paper 4813, Houston, 1984.
- (9) INGLIS, R.B., A 3-D analysis of the motion of a rigid ship in waves, Ph.D. Thesis, University College, London, 1980.
- (10) BOUGIS, J., Etude de la diffraction radiation dans le cas d'un flotteur indeformable animé par une houle sinusoidale de faible amplitude, Ph.D. Thesis Université de Nantes, 1980.
- (11) CHANG, M.S., Computation of three-dimensional ship motions with forward speed, Second International Conference on Numerical Ship Hydrodynamics, Berkeley, 1977.
- (12) EGGERS, K., Non-Kelvin dispersive waves around non-slender ships, Schiffstechnik, Bd. 8, 1981.
- (13) BABA, E., Wave resistance of ships in low speed, Mitsubishi Technical Bulletin, No. 109, 1976.

-
- (14) HERMANS, A.J., The wave pattern of a ship sailing at low speed, Report 84A, University o Delaware, 1980.
 - (15) BRANDSMA, F.J. and HERMANS, A.J., A quasi-linear free surface condition in slow ship theory, Schiffstechnik, April, 1985.
 - (16) WEHAUSEN, J.V. and LAITONE, E.V., Surface waves, Handbook of Physics, Vol.9, 1960.
 - (17) PINKSTER, J.A., Low frequency second order wave exciting forces on floating structures, Ph.D. Thesis, Report 650, MARIN (N.S.M.B), 1980.
 - (18) HUIJSMANS, R.H.M., Wave drift forces in current, 16th Conference on Naval Hydrodynamics, Berkeley, 1986.
 - (19) FALTINSEN, O., DAHLE, L. and SORTLAND, B., Slow drift damping and response of a moored ship in irregular waves, Offshore Mechanics and Architects Engineering Conference, Tokyo, 1986.
 - (20) SORTLAND, B., Force measurements in oscillating flow on ship sections and circular cylinders in a U-tube watertank, Report UR-86-52, NIT, 1986.
 - (21) FALTINSEN et al., Prediction of resistance and propulsion of a ship in a seaway, 13-th ONR Symposium, Tokyo, 1980.