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On the Low-Frequency Hydrodynamic Damping Forces Acting on Offshore Moored Vessels

by J.E.W. Wichers and R.M.H. Huijsmans, *Netherlands Ship Model Basin*

ABSTRACT

Moored floating structures for drilling, production-storage-offloading or other purposes are being installed in ever increasing water depths and in areas where the environmental conditions are also more severe. Such structures, moored permanently in high seas, have to survive safely the most severe weather conditions. Therefore it is of importance to understand the mechanisms which govern the motions and the mooring forces of these facilities.

In deep water the mooring systems inevitably have soft elasticity characteristics. With the increase in the elasticity of the mooring, the low frequency horizontal motions induced by low frequency second order wave drift forces also become larger. The low frequency resonant motion components completely dominate the horizontal motions and, consequently, also the mooring forces. In order to predict the amplitudes of the low frequency resonant motions the magnitude of the second order wave drift forces and the values of the low frequency hydrodynamic damping must be known.

The low frequency hydrodynamic potential damping due to the radiated waves is negligibly small. In general the low frequency damping is determined by viscous effects and a damping caused by the presence of waves. The last mentioned damping is called the wave damping. Dependent on the wave spectra the wave damping can significantly dominate the viscous damping contributions.

In this paper results are given of a study of the origin of the wave damping. For this purpose the second order wave drift forces acting on the moored vessel in head waves have been expanded to the low frequency surge displacement and surge velocity. The wave damping can be defined by taking into account the dependence on velocity of the second order wave drift forces. To verify the results, model experiments were carried out in which the velocity dependent second order wave drift forces were determined.

References and illustrations at end of paper.

INTRODUCTION

Floating structures mooored at sea are subjected to forces that tend to shift them from their desired position. For a given vessel the motions depend both on the mooring system and on the external forces acting on the vessel. The forces on the vessel caused by an irregular sea are of an irregular nature and may be split into two parts:

- first order oscillatory forces with wave frequency and,
- second order slowly varying forces with frequencies much lower than the wave frequency.

The first order oscillatory wave forces on a vessel cause the well-known ship motions, with frequencies equal to the frequencies present in the spectrum of the irregular waves.

The second order wave forces, also known as the wave drift forces, have been shown to be proportional to the square of the wave height, while the frequencies correspond to the frequencies present in the spectrum of the wave groups in the irregular waves, [1].

The magnitude of the second order, slowly varying forces is small compared to the magnitude of the first order, high frequency oscillatory wave forces. Considering a vessel moored in head waves the natural frequency of the system in longitudinal direction is very low. Since the damping at the low frequencies is very small the second order slowly varying forces cause large amplitude resonant motions. To predict the low frequency resonant amplitudes the magnitude of the damping must be known. At the low frequencies the potential damping due to wave radiation caused by the surge oscillations is negligibly small. The damping must be found in another direction.

In [2] results are presented of surge extinction tests carried out in still water and in a regular wave field with various periods and heights. The results revealed that besides a still water viscous damping also a quadratic wave damping transfer function exists due to the presence of the waves only.

Time domain simulations carried out with a mathematical model incorporating the above findings have clearly demonstrated the importance of the effect of the wave damping on the low frequency resonant surge motions, [3].

In order to understand the mechanism of the damping a study has been performed on the origin of the wave damping. For this purpose the multiple scale relation (frequency and motion scales) of the equation of motion in surge direction of a moored vessel in head waves has been considered, while further the wave exciting loads have been expanded to the surge displacement and surge velocity. Following the expansions of the motions and the wave exciting loads a more general description of the equation of motion has been established. In analyzing the low frequency part of the equation of motion it turns out that the wave damping as derived from the extinction tests seems to correspond to the derivative of the mean wave drift force at zero speed to the low frequency surge velocity. In order to verify the finding model experiments were carried out to determine the second order wave drift forces as a function of the vessel velocity (= added resistance). From the results the definition of the wave damping coefficient could be confirmed.

EQUATIONS OF MOTIONS

The high and low frequency motion components

As observed by Wichers, [3] the motions of a moored 200 kTDW tanker in irregular head waves consist of small amplitude high frequency surge, heave and pitch motions and large amplitude low frequency surge motions. The high frequency motions are related to the individual wave frequency components in the wave train. The low frequency surge motion is concentrated around the natural frequency of the moored vessel. In order to determine the amplitudes of the low frequency surge motions the damping at the natural surge frequency of the system has to be known.

To study the low frequency hydrodynamic damping forces acting on a structure use has been made of two different systems of axes as indicated in Figure 1. The system of axes Ox_1x_3 is fixed in space, with the Ox_1-axis in the still water surface and the Ox_3-axis coinciding with the vertical axis Gx_3 of the ship-fixed system of axes Gx_1x_3, at rest. We shall assume that the surge, heave and pitch motions can be decoupled into the following form:

$$x_1 = \varepsilon\, x_1^{(1)}(t/\eta,t) + \varepsilon^2\left(x_{11f}^{(2)}(t) + x_{1hf}^{(2)}(\frac{2t}{\eta},t)\right)$$

$$x_3 = \varepsilon\, x_3^{(1)}(t/\eta,t) + \varepsilon^2\left(x_{31f}^{(2)}(t) + x_{3hf}^{(2)}(\frac{2t}{\eta},t)\right)$$

$$x_5 = \varepsilon\, x_5^{(1)}(t/\eta,t) + \varepsilon^2\left(x_{51f}^{(2)}(t) + x_{5hf}^{(2)}(\frac{2t}{\eta},t)\right) \quad \ldots\ldots\ldots\ldots (1)$$

with ε and η are small parameters, viz.:

- ε relates to the wave steepness
- η considers the ratio between the two time-scales of the motions: the μ frequency range of the natural frequency of the system and the ω frequency range of the wave spectrum frequencies.

Further:

- $x_1^{(1)}$, $x_3^{(1)}$ and $x_5^{(1)}$ relate to the wave frequency surge, heave and pitch motions.
- $x_{11f}^{(2)}$, $x_{31f}^{(2)}$ and $x_{51f}^{(2)}$ stand for the large amplitude low frequency second order surge, heave and pitch motions.
- $x_{1hf}^{(2)}$, $x_{3hf}^{(2)}$ and $x_{5hf}^{(2)}$ represent the second order motions of which the frequency range is twice the wave frequency range.

Of the second order motions only the low frequency part will be considered and will be denoted as $\underline{x}^{(2)}$. Following the expansion of the motions and substitution into Newton's law we obtain for the surge direction:

$$M\left(\ddot{x}_1^{(2)} + \ddot{x}_1^{(1)}\right) = F_R\left(\underline{x}^{(1)}, x_1^{(2)}\right) + F_M + F_W \quad \ldots\ldots\ldots\ldots (2)$$

in which:

F_R = fluid reactive forces
F_M = restoring force due to the mooring system
F_W = wave exciting forces.

The high frequency fluid reaction forces can be written as a convolution integral as has been shown by Cummins, [4]. The low frequency fluid reaction forces can be separated into a reaction part of potential nature and a part of viscous origin. From these observations the equation of motion will be:

$$(M + a_{11}(\mu))\ddot{x}_1^{(2)} + \sum_{j=1,3,5} \left\{(M+m_{1j})\ddot{x}_j^{(1)} + \int_0^\infty K_{1j}(\tau)\dot{x}_j^{(1)}(t-\tau)\, d\tau\right\} = F_V(\dot{x}_1^{(2)}, \dot{x}_1^{(1)}) + - C_{11}(x_1^{(2)} + x_1^{(1)}) + F_W \quad \ldots\ldots\ldots\ldots (3)$$

In the left side of eq. (3) the fluid reactive force due to the low frequency surge motion is described by the term $a_{11}(\mu)$, the surge added mass at the natural frequency μ of the system. From the data given in Figure 6 it can be found that no potential damping due to wave radiation exists ($\mu \leq 0.08$ rad/s ; $b_{11}(\mu) \approx 0$). In the right hand side of eq. (3), $F_V(\dot{x}_1^{(2)}, \dot{x}_1^{(1)})$ describes the fluid reactive forces which are of viscous nature. For this term one may write:

$$F_V(\dot{x}_1^{(2)}, \dot{x}_1^{(1)}) = -b_{011}(\dot{x}_1^{(2)} + \dot{x}_1^{(1)}) + - b_{111}(\dot{x}_1^{(2)} + \dot{x}_1^{(1)})\,|\dot{x}_1^{(2)} + \dot{x}_1^{(1)}| \quad \ldots\ldots (4)$$

The second term at the right hand side of eq. (3) represents the restoring force caused by the mooring system. The coefficient C_{11} stands for the spring constant in longitudinal direction. For the wave exciting forces the following expansion is used:

$$F_W = \varepsilon F^{(1)} + \varepsilon^2 F^{(2)} \quad \ldots\ldots\ldots\ldots (5)$$

where $F^{(1)}$ is the first order wave exciting force oscillating at wave frequencies. $F^{(2)}$ is the low frequency second order wave drift force on a floating body in irregular waves, oscillating at wave group frequencies, disregarding the force oscillations at the double of the wave frequencies. The first order wave exciting forces are only dependent on the incoming wave height, wave period, water depth and geometry of the body. These first order wave forces can be calculated by a linear 3-D diffraction program, see [5]. The wave force $F^{(2)}$ on a stationary floating body in waves is calculated by a pressure integration technique as developed by Pinkster, [1]. In his study it is assumed that the floating body only performs small amplitude high frequency motions around the mean position. Following the conditions of the mentioned computations the first order wave exciting forces and the second order wave drift forces should be written as follows:

$$F^{(1)} = F^{(1)}(x_1^{(2)} = 0,t) \quad \ldots\ldots\ldots (6)$$

$$F^{(2)} = F^{(2)}(x_1^{(2)} = 0,\ \underline{x}^{(1)},t) \quad \ldots\ldots (7)$$

As mentioned before in reality, however, the vessel performs small amplitude high frequency motions while travelling with large amplitude low frequency surge oscillations through the wave field. This observation implies that the wave exciting load formulation has to be adopted for a low frequency displacement and velocity dependency in the wave field.

Displacement dependency

Neglecting, however, the influence of the low frequency surge velocities the wave exciting loads will be dependent on the low frequency displacement of the vessel in the irregular wave field only. In the following we shall therefore assume:

$$F^{(1)} = F^{(1)}(x_1^{(2)},t) \quad \ldots\ldots\ldots\ldots (8)$$

$$F^{(2)} = F^{(2)}(x_1^{(2)},\ \underline{x}^{(1)},t) \quad \ldots\ldots\ldots (9)$$

with: $\underline{x}^{(1)} = [x_1,\ x_3,\ x_5]^T$

The computation procedure in the time domain of $F^{(1)}(x_1^{(2)},t)$ and $F^{(2)}(x_1^{(2)},\ \underline{x}^{(1)},t)$ has been shown by Wichers and Van de Boom, [6]. The computation of the wave loads has been formulated using convolution integrals based on the wave height at the actual (instantaneous) position of the vessel in the wave field. The procedure is briefly described below.

$$F^{(1)}(x_1^{(2)},t) = \int_0^\infty g^{(1)}(\tau)\ \zeta(x_1^{(2)},t-\tau)d\tau \quad \ldots\ldots\ldots\ldots (10)$$

$$F^{(2)}(x_1^{(2)},\underline{x}^{(1)},t) = \int_0^\infty\int_0^\infty g^{(2)}(\tau_1,\tau_2)\ .$$

$$\zeta(x_1^{(2)},t-\tau_1)\ \zeta(x_1^{(2)},t-\tau_2)\ d\tau_1\ d\tau_2 \quad \ldots (11)$$

The wave elevation $\zeta(x_1^{(2)},t)$ at the actual location of the vessel may be obtained from the wave elevation at the space fixed point 0 being $\zeta(x_1^{(2)}=0,t)$:

$$\zeta(x_1^{(2)},t) = \int_0^\infty w(x_1^{(2)},\tau)\ \zeta(x_1^{(2)}=0,t-\tau)d\tau \quad \ldots\ldots\ldots\ldots (12)$$

in which:

$$w(x_1^{(2)},\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} W(\omega)\ e^{-i\omega\tau}\ d\omega \quad \ldots\ldots (13)$$

$$W(\omega) = \frac{F\{\zeta(x_1^{(2)},t)\}}{F\{\zeta(x_1^{(2)}=0,t)\}} \quad \ldots\ldots\ldots\ldots (14)$$

in which:

F{ } denotes Fourier transforms.

The linear and quadratic impulse response functions $g^{(1)}$ and $g^{(2)}$ are found from the Fourier transform of the corresponding frequency domain transfer functions $G^{(1)}(x_1^{(2)}=0,\omega)$ and $G^{(2)}(x_1^{(2)}=0,\underline{x}^{(1)},\omega_1,\omega_2)$:

$$g^{(1)}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G^{(1)}(x_1^{(2)}=0,\omega)\ e^{-i\omega\tau}d\omega \quad \ldots\ldots\ldots\ldots (15)$$

$$g^{(2)}(\tau_1,\tau_2) =$$

$$= \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G^{(2)}(x_1^{(2)}=0,\underline{x}^{(1)},\omega_1,\omega_2)\ .$$

$$e^{-i(\omega_1\tau_1-\omega_2\tau_2)}\ d\omega_1 d\omega_2 \quad \ldots\ldots\ldots (16)$$

By means of the formulations the effect of the low frequency surge displacement of the vessel in the wave field on the wave exciting forces is taken into account. This is a first approximation of the actual problem since the performance of large amplitude low frequency surge oscillations of the tanker implies low frequency surge velocities of the body.

Displacement and velocity dependency

If besides the low frequency displacement also the low frequency velocity of the vessel is taken into account, speed effects will be introduced into the hydrodynamics. Due to the low frequency oscillating speed not only the pressures on the hull but also the oscillation frequency of the vessel will be affected. A similar problem can be found in the case of wave resistance at forward speed. The speed effect may influence both the wave exciting loads and the high frequency fluid reactive forces.

For the wave exciting loads we shall therefore assume:

$$F^{(1)} = F^{(1)}(x_1^{(2)},\ \dot{x}_1^{(2)},t) \quad \ldots\ldots\ldots (17)$$

$$F^{(2)} = F^{(2)}(x_1^{(2)},\dot{x}_1^{(2)},\underline{x}^{(1)},t) \quad \ldots\ldots\ldots (18)$$

By applying the Taylor expansion of the wave forces to the low frequency displacement and velocity up to the first order variations we obtain, for the first order wave force:

$$F^{(1)}(x_1^{(2)},\dot{x}_1^{(2)},t) = F^{(1)}(0,0,t) +$$

$$+ x_1^{(2)}\frac{\partial F^{(1)}(0,0,t)}{\partial x_1^{(2)}} + \dot{x}_1^{(2)}\frac{\partial F^{(1)}(0,0,t)}{\partial \dot{x}_1^{(2)}} \quad \ldots\ldots\ldots\ldots (19)$$

and for second order wave force:

$$F^{(2)}(x_1^{(2)},\dot{x}_1^{(2)},\underline{x}^{(1)},t) = F^{(2)}(0,0,\underline{x}^{(1)}t) +$$

$$+ x_1^{(2)}\frac{\partial F^{(1)}(0,0,\underline{x}^{(1)},t)}{\partial \dot{x}_1^{(2)}} +$$

$$+ \dot{x}_1^{(2)}\frac{\partial F^{(2)}(0,0,\underline{x}^{(1)},t)}{\partial \dot{x}_1^{(2)}} \quad \ldots\ldots\ldots\ldots (20)$$

Taking into account the high frequency hydrodynamic reaction forces influenced by the speed effect and substituting eq. (19) and (20) into eq. (3) the approximation of the assumed general equation of motion in surge direction of the vessel moored in irregular waves can be written as follows:

$$\left[M+a_{11}(\mu)\right]\ddot{x}_1^{(2)} + \sum_{j=1,3,5} \cdot$$

$$\cdot \underbrace{\left\{M\ddot{x}_j^{(1)} + m_{1j}(\dot{x}_1^{(2)})\ddot{x}_j^{(1)} + \int_0^\infty K_{1j}(\dot{x}_1^{(2)},\tau)\dot{x}_j(t-\tau)d\tau\right\}}_{\text{high frequency fluid reaction forces including speed effect}} =$$

$$= - b_{011}(\dot{x}_1^{(2)} + \dot{x}_1^{(1)}) - b_{111}(\dot{x}_1^{(2)} + \dot{x}_1^{(1)}) \cdot$$

$$\underbrace{\cdot \left|\dot{x}_1^{(2)} + \dot{x}_1^{(1)}\right| + c_{11}(x_1^{(2)} + x_1^{(1)}) + \underbrace{F^{(1)}(0,0,t) + x_1^{(2)}\frac{\partial F^{(1)}(0,0,t)}{\partial x_1^{(2)}}}_{\text{corresponding to eq. (10)}} + \dot{x}_1^{(2)}\frac{\partial F^{(1)}(0,0,t)}{\partial \dot{x}_1^{(2)}}}_{\textbf{High frequency}} +$$

$$\underbrace{+ \underbrace{F^{(2)}(0,0,\underline{x}^{(1)},t) + x_1^{(2)}\frac{\partial F^{(2)}(0,0,\underline{x}^{(1)},t)}{\partial x_1^{(2)}}}_{\text{corresponding to eq. (11)}} + \dot{x}_1^{(2)}\frac{\partial F^{(2)}(0,0,\underline{x}^{(1)},t)}{\partial \dot{x}_1^{(2)}}}_{\textbf{Low frequency}}$$

$$\ldots\ldots\ldots\ldots \quad (21)$$

In eq. (21) both the high and low frequency force components are incorporated. In order to arrive at the low frequency hydrodynamic damping forces acting on the moored structure the force components which will contribute to the low frequency motions have to be considered first.

It is obvious that the high frequency fluid reaction forces will be independent of the low frequency surge displacements. The first order wave exciting force $F^{(1)}(x_1^{(2)},\dot{x}_1^{(2)},t)$ is also assumed to be independent of the low frequency of surge displacement. The force component $\dot{x}_1^{(2)}\frac{\partial F^{(1)}(0,0,t)}{\partial \dot{x}_1^{(2)}}$ can be deleted based on the following reasoning:
The frequency range in which the coefficient $\frac{\partial F^{(1)}(0,0,t)}{\partial \dot{x}_1^{(2)}}$ oscillates will be $\omega \pm \mu$. Because μ is the natural frequency of the system the oscillating coefficient will be in the high (wave) frequency range. This means that the force component will not contribute to the low frequency motions. After inspection of the terms of eq. (21) the equation of the low frequency motion in irregular waves can be written as follows:

$$\left[M+a_{11}(\mu)\right]\ddot{x}_1^{(2)} = - b_{011} \cdot \dot{x}_1^{(2)} - b_{111}(\dot{x}_1^{(2)} + \dot{x}_1^{(1)})\left|\dot{x}_1^{(2)} + \dot{x}_1^{(1)}\right| - c_{11}x_1^{(2)} + F^{(2)}(0,0,\underline{x}^{(1)},t) + x_1^{(2)}\frac{\partial F^{(2)}(0,0,\underline{x}^{(1)},t)}{\partial x_1^{(2)}} + \dot{x}_1^{(2)}\frac{\partial F^{(2)}(0,0,\underline{x}^{(1)},t)}{\partial \dot{x}_1^{(2)}} \quad \ldots\ldots\ldots (22)$$

In the right hand side of eq. (22) three low frequency coefficients can be recognized. The first two terms are assumed to be of viscous nature while the last term relates to a potential effects.

In order to analyze and verify the separate terms of eq. (22) model tests have been carried out. The following experiments were carried out:

1. Motion decay test:
   - in still water
   - in regular head waves with various wave heights and periods.
2. Towing tests at low vessel speed:
   - in still water
   - in regular head waves with various wave heights and periods.

EXPERIMENTS

Test set-up

Use was made of a model of a fully loaded 200 kDWT tanker (scale 1:82,5). The particulars of the vessel are given in Table 1. The body plan is shown in Figure 2. The test set-up of the mooring arrangement for the model experiments is shown in Figure 3. For the extinction tests a linear mooring system was employed for the tanker. The spring constant was 16.0 tf/m. The extinction tests were carried out in the Wave and Current Basin of the Netherlands Ship Model Basin, measuring 60x40 m and having a water depth of 1 m. The towing tests were carried out in the Seakeeping basin of the Netherlands Ship Model Basin, having a water depth of 2.5 m, a length of 100 m and a width of 24 m. For the towing tests the mooring system, consisting of linear springs, was connected to the towing carriage. The spring constant amounted to 257.4 tf/m. During the towing tests the model was kept in longitudinal direction by means of a light weight trim device connected at the F_{pp} and A_{pp}.

Measurements

During the tests the surge, heave and pitch motions and the longitudinal mooring forces were measured. The surge and heave motions were measured in the centre of gravity (G) by means of an optical tracking device. The pitch motion was measured by means of a gyroscope. The sign convention is given in Figure 3. The mooring lines were connected to ship-bound force transducers. The longitudinal towing or mooring forces were measured by means of the force transducers. All measurements were recorded on magnetic tape to facilitate the data analysis. All data were scaled to prototype according to Froude's law of similitude.

MEAN WAVE DAMPING QUADRATIC TRANSFER FUNCTION (EXTINCTION TESTS)

From results of earlier extinction tests in regular waves an additional low frequency hydrodynamic damping force due to the presence of the waves only was identified, [2]. In the present experimental study more accurate and comprehensive extinction experiments were carried out for confirmation. In combination with eq. (22) the results will be analyzed.

Still water

Applying eq. (22) to the condition of extinction in still water the equation of motion reduces to:

$$\{M+a_{11}(\mu)\}\ddot{x}^{(2)} = - b_{011}\,\dot{x}_1^{(2)} - b_{111}(\dot{x}_1^{(2)} + \dot{x}_1^{(1)})\,|\dot{x}_1^{(2)} + \dot{x}_1^{(1)}| - c_{11}\,x_1^{(2)} \quad \ldots . (23)$$

The results of the experimental extinction test are shown in Figure 4 and Figure 5. From the result of the extinction test it follows that for the large amplitude surge motions the viscous damping force is approximately linearly proportional to the low frequency velocity:

$$b_{011}(\mu = 0{,}024 \text{ rad/s}) = 18.2 \text{ tf.s/m}$$

$$b_{111} = 0 \text{ tf.s/m}$$

(μ = natural frequency of the moored system)

As is indicated in Figure 6 the still water viscous damping coefficient for low frequency surge motion is dominant. The potential damping due to radiated waves is negligibly small at low frequencies.

Regular waves

Eq. (22) applied to the condition of extinction in regular waves will read:

$$\{M+a_{11}(\mu)\}\ddot{x}_1^{(2)} = - b_{011}\,\dot{x}_1^{(2)} - b_{111}(\dot{x}_1^{(2)} + \dot{x}_1^{(1)})\,|\dot{x}_1^{(2)} + \dot{x}_1^{(1)}| - c_{11}\,x_1^{(2)} + \overline{F}^{(2)}(0,0,\underline{x}^{(1)},t) + x_1^{(2)}\,\frac{\partial\overline{F}^{(2)}(0,0,\underline{x}^{(1)},t)}{\partial x_1^{(2)}} + \dot{x}_1^{(2)}\,\frac{\partial\overline{F}^{(2)}(0,0,\underline{x}^{(1)},t)}{\partial x_1^{(2)}} \quad \ldots\ldots\ldots (24)$$

In a regular wave with wave frequency ω it is known that:

$$\overline{F}^{(2)}(0,0,\underline{x}^{(1)},t) = \text{constant} \quad \ldots\ldots (25)$$

and since the mean wave drift force is independent of the position in the regular waves it is true that:

$$\frac{\partial\overline{F}^{(2)}(0,0,\underline{x}^{(1)},t)}{\partial x_1^{(2)}} = 0 \quad \ldots\ldots\ldots (26)$$

Further it is assumed that the damping coefficient will be constant in the regular wave:

$$\frac{\partial\overline{F}^{(2)}(0,0,\underline{x}^{(1)},t)}{\partial\dot{x}_1^{(2)}} = \text{constant} \quad \ldots\ldots (27)$$

Denoting $\dfrac{\partial\overline{F}^{(2)}(0,0,\underline{x}^{(1)},t)}{\partial\dot{x}_1^{(2)}} = -\,b_{211}$ and taking into account eq. (25) and (26), eq. (24) reduces to:

$$\{M+a_{11}(\mu)\}\ddot{x}_1^{(2)} = -(b_{011} + b_{211})\dot{x}_1^{(2)} - b_{111}(\dot{x}_1^{(2)} + \dot{x}_1^{(1)})\,|\dot{x}_1^{(2)} + \dot{x}_1^{(1)}| - c_{11}\,\dot{x}_1^{(2)} + \overline{F}^{(2)}(0,0,\underline{x}^{(1)},t) \quad \ldots\ldots (28)$$

Results of the experimental extinction tests are given in Figures 4 and 5. Figure 5 shows that for both wave amplitudes applied (constant wave frequency ω) the total low frequency damping force is linearly proportional to the low frequency surge velocity. This observation leads to the conclusion that the contribution of the quadratic viscous damping to the total damping is negligibly small ($b_{111} \approx 0$). This observation seems to contradict the results of the study of Kato and Kitoshita, [7]. In their study a free low frequency oscillating system in surge direction with a non-linear (quadratic) damping was excited with and without a high frequency force. Due to the high frequency disturbance the non-linear damping significantly influences the low frequency motion. The deviation probably may be found in the assumed coefficients in their equation of motion.

Neglecting the quadratic viscous damping force, eq. (28) reduces to:

$$\{M+a_{11}(\mu)\}\ddot{x}_1^{(2)} = -(b_{011} + b_{211})\dot{x}_1^{(2)} - c_{11}\,x_1^{(2)} + \overline{F}^{(2)}(0,0,\underline{x}^{(1)},t) \quad \ldots\ldots (29)$$

Based on the linearity of damping coefficients as shown in Figure 5 the viscous damping coefficient b_{011} can be separated from the total damping coefficient. The damping coefficint b_{211} as a function of the square of the wave height is shown in Figure 7. This damping coefficient appears to be linearly proportional to the square of the wave height. Since b_{211} is related to the second order wave drift force the damping coefficient b_{211} is considered to be of potential nature. The coefficient b_{211} is called the mean wave damping coefficient. The mean wave damping quadratic transfer function as a function of the wave frequency can be determined from the extinction experiments carried out for various wave heights and frequencies:

$$\frac{b_{211}}{\zeta_a^2} = -\,\frac{\dfrac{\partial\overline{F}^{(2)}(0,0,\underline{x}^{(1)},t)}{\partial\dot{x}_1^{(2)}}}{\zeta_a^2} \quad \ldots\ldots\ldots (30)$$

The transfer function is shown in Figure 8. Based on the assumptions in eq. (22) it appears that in a regular wave the wave damping coefficient represents the derivative of the mean longitudinal wave drift force at zero speed to the low frequency vessel velocity in surge direction. To prove this hypothesis the dependency of the mean wave drift force on the vessel has to be known. For this purpose steady towing tests for vessel speeds around zero speed were carried out.

VELOCITY DEPENDENT WAVE DRIFT FORCE QUADRATIC TRANSFER FUNCTION (TOWING TESTS)

Still water

The towing tests in still water were carried out at various steady speeds. The towing directions were both backward and forward. Following eq. (22) and taking for the low frequency oscillation speed the steady speed $\dot{x}_1^{(2)} = U$ the mean towing force $\overline{F}_s$ can be described as follows:

$$\overline{F}_S = - b_{011}U - b_{111}U^2 =$$
$$= -\tfrac{1}{2}\rho \; L \; . \; T \; . \; C(U) \; . \; U^2 \quad \ldots\ldots\ldots (31)$$

in which:
$C(U)$ = measured resistance coefficient,
ρ = specific density of seawater,
L = length of ship between perpendiculars,
T = draft of the ship.

The measured resistance coefficient $C(U)$ is shown in Figure 9 as a function of the vessel velocity.

Regular waves

The towing tests in regular waves were carried out under the same speed conditions as for the still water towing tests (except for the 5 knots speed). Following eq. (20), (22) and (31) and assuming that $\overline{F}_T$ represents the total mean towing force for the steady state condition, the total mean towing force will be:

$$\overline{F}_T = -[-\tfrac{1}{2}\rho \; L \; . \; T \; . \; C(U)(U^2 + \overline{(\dot{x}_1^{(1)}\;)^2})] +$$
$$- \overline{F}^{(2)}(\dot{x}_1^{(2)},U,\underline{x}^{(1)},t) \quad \ldots\ldots\ldots (32)$$

Since in a regular wave $\overline{F}^{(2)}(\dot{x}_1^{(2)},U,\underline{x}^{(1)},t)$ is independent of $\dot{x}_1^{(2)}$ and for the viscous resistance force formulation $U^2 >> (\dot{x}_1^{(1)})^2$ eq. (32) can be simplified to:

$$\overline{F}_T = + \tfrac{1}{2}\rho \; L \; . \; T \; . \; C(U) \; . \; U^2 - \overline{F}^{(2)}(U,\underline{x}^{(1)},t) \quad \ldots\ldots\ldots\ldots (33)$$

$\overline{F}^{(2)}(U,\underline{x}^{(1)},t)$ actually represents the velocity dependent mean wave drift force or the added resistance force at vessel speed U. From the experiments carried out for various wave heights, wave periods and vessel speeds the mean wave drift force can be established as a function of the vessel speed (being the difference between the measured total towing force and the appropriate towing force in still water).

The results presented as the velocity dependent mean wave drift force quadratic transfer functions are shown in Figure 10. The solid line represents the zero speed mean wave drift force quadratic transfer function. The dotted lines display the transfer functions for the other speeds.

From a theoretical point of view these transfer functions can be applied directly to current (zero vessel speed) if the horizontal ω-scale is transformed into the ω_e-scale according to the following relation:

$$\omega_e = \omega + \frac{2\pi}{\lambda} \; . \; U \quad \ldots\ldots\ldots\ldots (33)$$

in which:
ω_e = the wave frequency observed in a current speed U,
λ = wave length related to the wave frequency ω.

The dependency of the mean wave drift forces on the speed of the vessel can be clarified more explicitly if we examine the speed dependency per wave frequency. This is shown in Figure 11.

From this figure the mean wave drift force is shown to be a linear function of the vessel speed for all wave frequencies.

The coefficients $\dfrac{\partial\overline{F}^{(2)}(0,0,\underline{x}_1^{(1)},t)}{\partial\dot{x}_1^{(2)}}$ as derived from Figure 11 are plotted in Figure 8. From these experimental findings one may conclude that the expansions used in eq. (21) holds true for this kind of configurations and that the derivative of the mean drift force quadratic transfer function to the vessel speed may be defined as the mean wave damping quadratic transfer function, or:

$$\frac{b_{211}}{\zeta_a^2} = - \frac{\dfrac{\partial\overline{F}^{(2)}(0,0,\underline{x}^{(1)},t)}{\partial\dot{x}_1^{(2)}}}{\zeta_a^2} \quad \ldots\ldots\ldots (34)$$

CALCULATION OF THE WAVE DAMPING QUADRATIC TRANSFER FUNCTION

In order to calculate the wave damping quadratic transfer function (or the velocity dependent wave drift force) it will be necessary to obtain an accurate motion response function of the floating body, where the speed effect is taken into account.

By a speed correction model it is possible to calculate an accurate motion response function. The results of the computations for the motion response have to be incorporated into a wave drift force formulation which embodies the effect of forward speed. Work is underway to develop this computation procedure.

CONCLUSIONS

The low frequency hydrodynamic damping forces acting on a tanker moored in irregular head waves can be separated in a viscous damping force and in a wave damping force.

The viscous damping force is caused by the interaction of the vessel and the fluid (still water) and appears to be linearly proportional to the low frequency velocity of the vessel.

The wave damping force is caused by the presence of the waves only and appears to be linearly proportional to the low frequency velocity of the vessel. The proportionality coefficient of this relation is dependent on the wave frequency. The wave damping force appears to be linearly proportional to the square of the wave height. As a consequence, the coefficient can be expressed as a mean wave damping quadratic transfer function

The origin of the wave damping is related to the low frequency velocity dependency of the mean second order wave drift force. For this reason the wave damping force can be considered as a reaction force.

The mean wave damping quadratic transfer function can be defined as being the derivative of the mean second order wave drift force quadratic transfer function to the low frequency velocity. The influence of the low vessel velocity on the second order wave drift force is relatively large.

NOMENCLATURE

- a_{11} = low frequency added mass in surge direction
- b_{11} = low frequency potential damping coefficient in surge direction
- b_{011} = low frequency (linear) viscous damping coefficient in surge direction
- b_{111} = low frequency (quadratic) viscous damping coefficient in surge direction
- b_{211} = low frequency wave damping
- C = resistance coefficient in surge direction
- c_{11} = spring constant in surge direction
- $F^{(1)}$ = first order wave force in surge direction
- $F^{(2)}$ = second order wave force in surge direction
- F_M = restoring force due to the mooring system
- F_R = fluid reactive force
- F_V = viscous damping force
- F_W = wave exciting loads
- G = centre of gravity
- $G^{(1)}$ = first order wave force linear transfer function in surge direction
- $G^{(2)}$ = second order wave force quadratic transfer function in surge direction
- $g^{(1)}$ = linear impulse response function
- $g^{(2)}$ = linear impulse response function
- i = $\sqrt{-1}$
- k_{ij} = retardation function in i-direction due to a motion in j-direction
- L = length of vessel between perpendiculars
- M = mass of the vessel
- m_{ij} = frequency independent added mass in i-direction due to a motion in j-direction
- O = origin of space fixed frame reference
- t = time
- T = draft of vessel or wave period
- U = vessel speed
- W = wave height transfer function
- w = impulse response function
- x_1 = surge motion
- x_3 = heave motion
- x_5 = pitch motion
- $\underline{x}^{(1)}$ = high frequency motion sector
- $x_1^{(2)}$ = low frequency surge motion
- ε = small parameter
- ζ = wave elevation
- ζ_a = wave amplitude
- η = small parameter
- λ = wave length
- μ = natural frequency of the system in surge direction
- ρ = specific density of seawater
- τ = time lag
- $\omega=2\pi/T$ = wave frequency
- ω_e = encounter frequency

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TABLE 1—MAIN PARTICULARS OF THE TANKER

Designation	Symbol	Unit	200 kTDW tanker
Length between perpendiculars	L_{pp}	m	310.00
Breadth	B	m	47.20
Draft even keel	T	m	18.90
Displacement volume	∇	m^3	234,994
Depth	H	m	20.70
Centre of gravity above keel	$\overline{KG}$	m	13.32
Longitudinal radius of gyration	k_{yy}	m	77.47
Block coefficient	C_B	-	0.850
Midship section coefficient	C_M	-	0.995
Waterline coefficient	C_W	-	0.890
Pitch period	T_Θ	s	10.2
Heave period	T_z	s	11.4

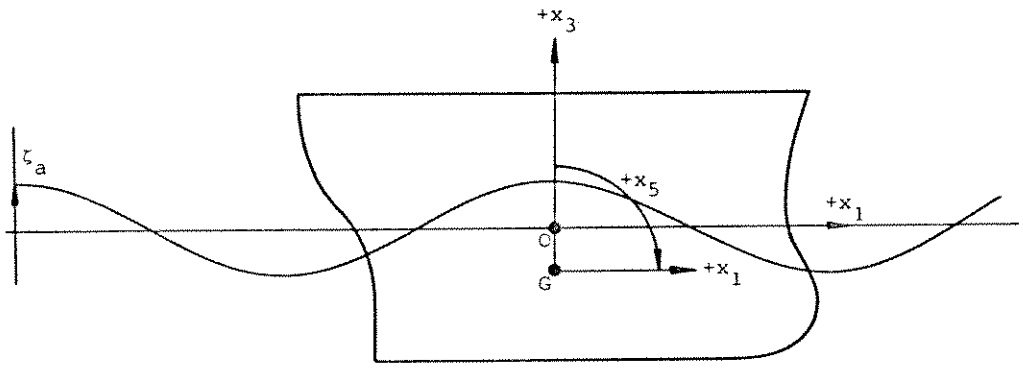


Fig. 1—Systems of coordinates.

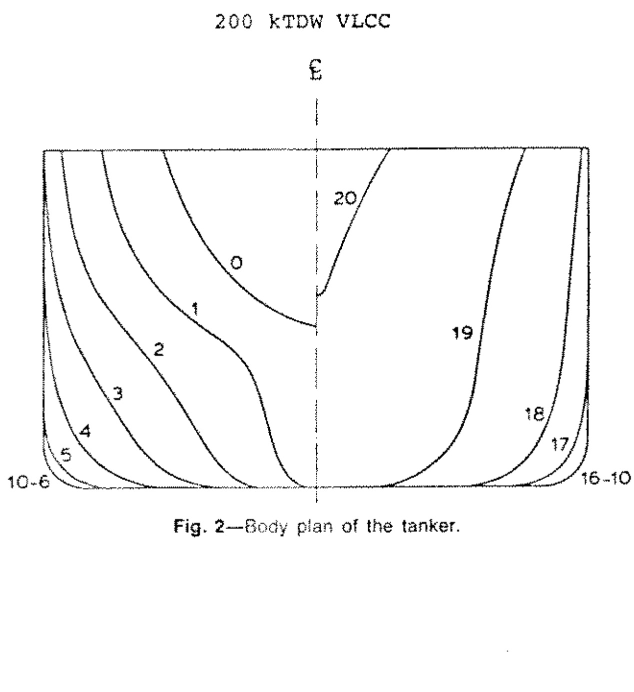


Fig. 2—Body plan of the tanker.

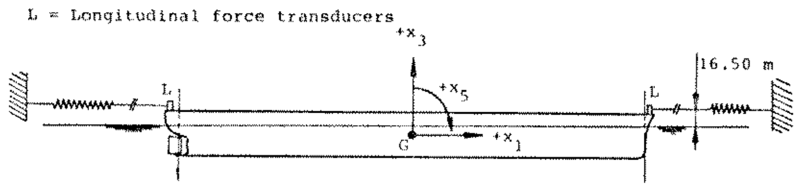
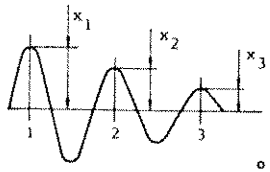


Fig. 3—Test setup.



$$b = \frac{\sqrt{C_{11}(M+a_{11}(\mu))}}{\pi} \cdot \frac{\ln x_1 - \ln x_{N+1}}{N}$$

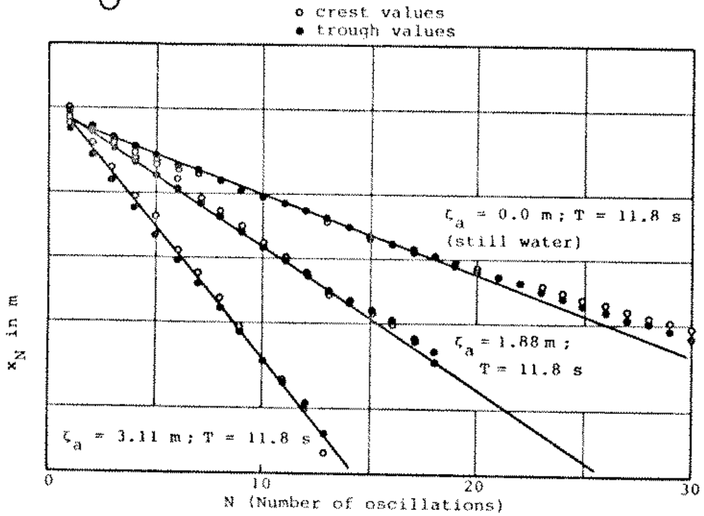


Fig. 5—Determination of the damping coefficients in still water and in regular waves.

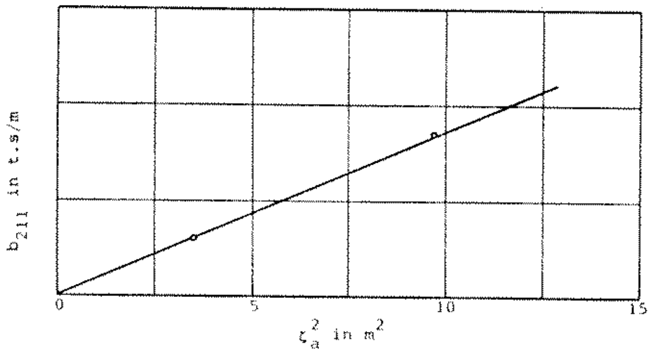


Fig. 7—Wave damping coefficient related to the square of the wave height.

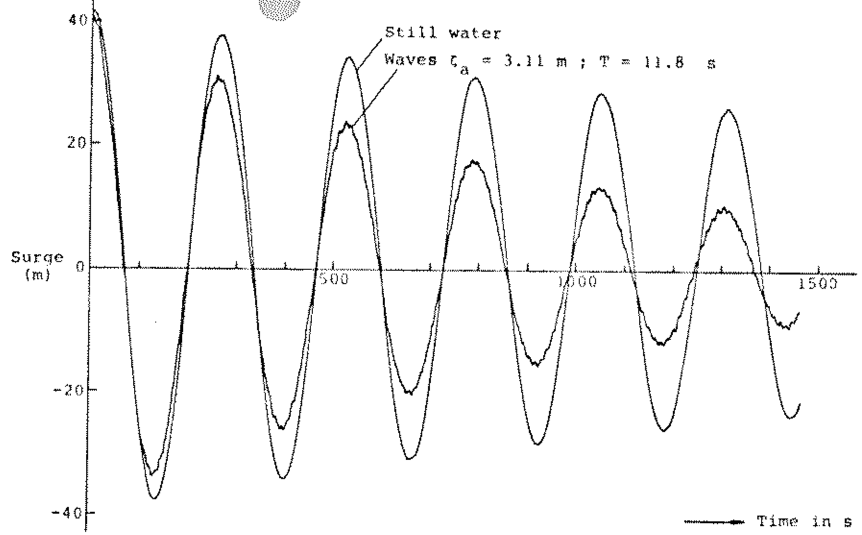


Fig. 4—Registration of extinction test in still water and in regular waves.

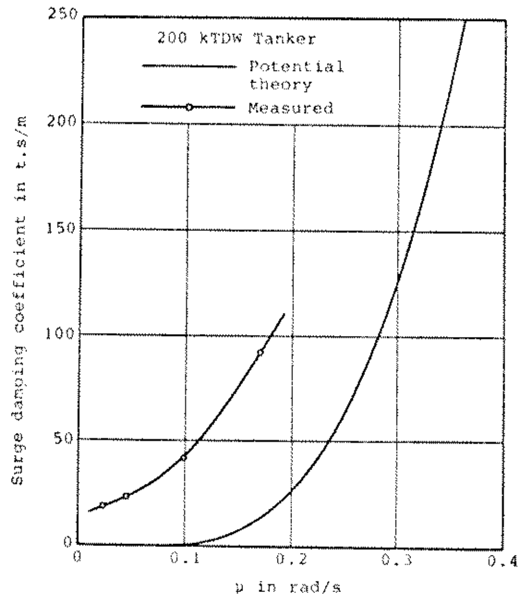


Fig. 6—Calculated and measured damping coefficients in still water derived from Ref. 2.

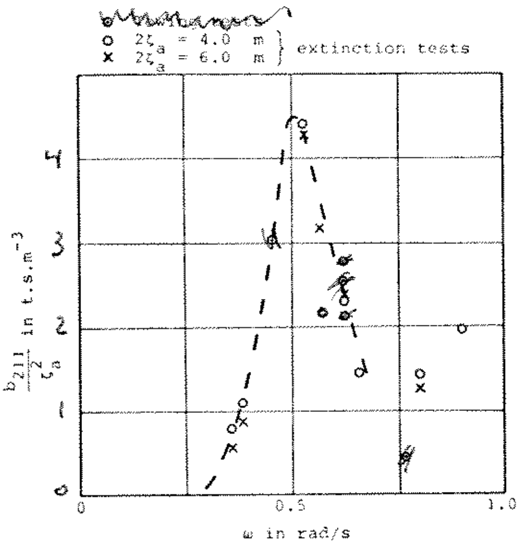


Fig. 8—The mean wave damping quadratic transfer function derived from extinction ~~and towing tests~~

m 82.5m WD

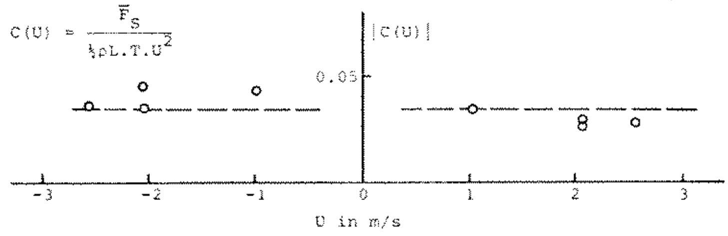


Fig. 9—Resistance coefficients measured during towing tests in still water.

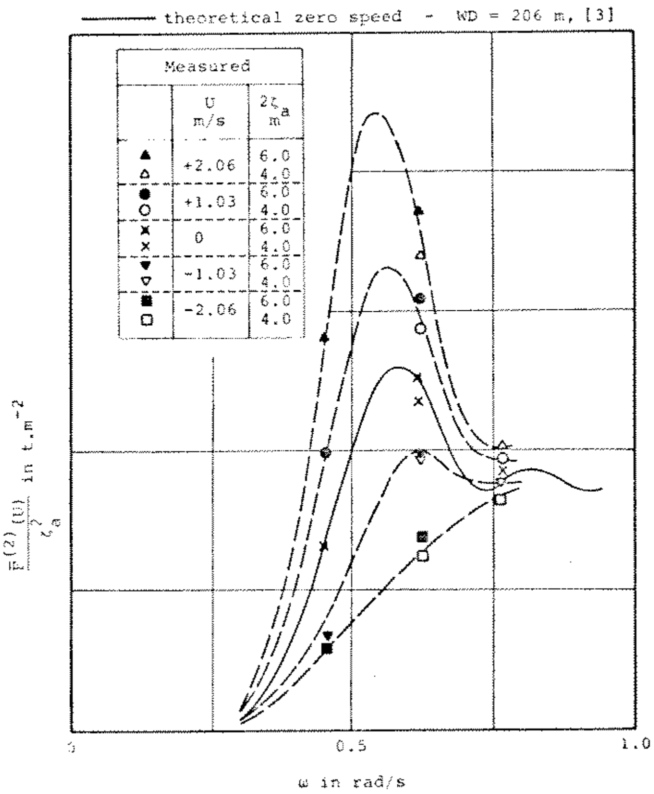


Fig. 10—Wave drift-force quadratic transfer function as functions at low vessel speed in regular head waves. (Wave frequency ω is with regard to the earth.)

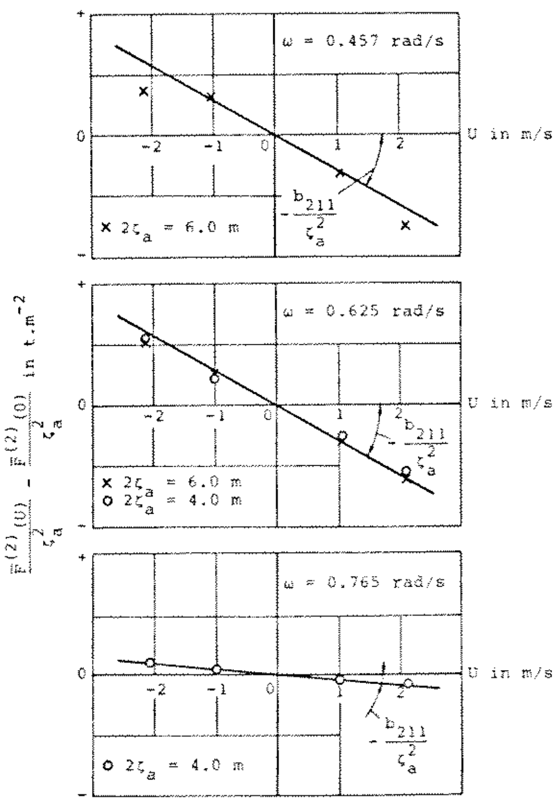


Fig. 11—Wave damping coefficients derived from towing tests for low vessel speeds.